Definition & Examples

Piezoelectricity is a coupling between a material's mechanical and electrical behaviors. In the simplest of terms, when a piezoelectric material is squeezed, an electric charge collects on its surface. Conversely, when a piezoelectric material is subjected to a voltage drop, it mechanically deforms.

Many crystalline materials exhibit piezoelectric behavior. A few materials exhibit the phenomenon strongly enough to be used in applications that take advantage of their properties. These include quartz, Rochelle salt, lead titanate zirconate ceramics (e.g. PZT-4, PZT-5A, etc.), barium titanate, and polyvinylidene flouride (a polymer film).

On a nanoscopic scale, piezoelectricity results from a nonuniform charge distribution within a crystal's unit cells. When such a crystal is mechanically deformed, the positive and negative charge centers displace by differing amounts. So while the overall crystal remains electrically neutral, the difference in charge center displacements results in an electric polarization within the crystal. Electric polarization due to mechanical input is perceived as piezoelectricity.

From an engineering or modeling point of view, piezoelectricity results in a change to a material's constitutive properties. Many finite element codes include piezoelectric modeling capability.

History

Piezoelectricity was discovered by Jacques and Pierre Curie in the 1880's during experiments on quartz.

Applications

Applications where strongly-piezoelectric materials are used include buzzers inside pagers and cell phones, shakers inside ultrasonic cleaners, spark generators inside electronic igniters, and strain sensors inside pressure gages.

Piezoelectric materials also make inexpensive but fantastically accurate "clocks". For example, the element keeping track of time inside a quartz watch is literally a small piece of vibrating quartz. Its vibration period is stable to more than one part per million as a result of its piezoelectic properties.

Hooke's Law and Dielectrics

What is a constitutive equation? For mechanical problems, a constitutive equation describes how a material strains when it is stressed, or vice-versa. Constitutive equations exist also for electrical problems; they describe how charge moves in a (dielectric) material when it is subjected to a voltage, or vice-versa.

Engineers are already familiar with the most common mechanical constitutive equation that applies for everyday metals and plastics. This equation is known as Hooke's Law and is written as:

$$\mathbf{S} = \mathbf{s} \cdot \mathbf{T}$$

In words, this equation states: **Strain = Compliance × Stress**.

However, since piezoelectric materials are concerned with electrical properties too, we must also consider the constitutive equation for common dielectrics:

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E}$$

In words, this equation states: **ChargeDensity = Permittivity × ElectricField**.

Coupled Equation

Piezoelectric materials combine these two seemingly dissimilar constitutive equations into one *coupled* equation, written as:

$$\mathbf{S} = \mathbf{s}_{\mathbf{E}} \cdot \mathbf{T} + \mathbf{d}^{t} \cdot \mathbf{E}$$
$$\mathbf{D} = \mathbf{d} \cdot \mathbf{T} + \boldsymbol{\varepsilon}_{\mathbf{T}} \cdot \mathbf{E}$$

The piezoelectric coupling terms are in the matrix $\mathbf{d}$.

In order to describe or model piezoelectric materials, one must have knowledge about the material's mechanical properties (compliance or stiffness), its electrical properties (permittivity), and its piezoelectric coupling properties.

Matrix Subscript Definitions

The subscripts in piezoelectric constitutive equations have very important meanings. They describe the conditions under which the material property data was measured.

For example, the subscript $\mathbf{E}$ on the compliance matrix $\mathbf{s}_{\mathbf{E}}$ means that the compliance data was measured under at least a constant, and preferably a zero, electric field.

Likewise, the subscript $\mathbf{T}$ on the permittivity matrix $\varepsilon_{\mathbf{T}}$ means that the permittivity data was measured under at least a constant, and preferably a zero, stress field.

Piezoelectric Constitutive Equation

Piezoelectricity is described mathematically within a material's constitutive equation, which defines how the piezoelectric material's stress (**T**), strain (**S**), charge-density displacement (**D**), and electric field (**E**) interact.

The piezoelectric constitutive law (in Strain-Charge form) is:

$$\mathbf{S} = \mathbf{s_E} \cdot \mathbf{T} + \mathbf{d}^t \cdot \mathbf{E}$$

$$\mathbf{D} = \mathbf{d} \cdot \mathbf{T} + \boldsymbol{\varepsilon}_\mathbf{T} \cdot \mathbf{E}$$

The matrix **d** contains the piezoelectric coefficients for the material, and it appears twice in the constitutive equation (the superscript *t* stands for matrix-transpose).

For a more detailed explanation of the constitutive variables, see the Symbol Definition page. For more on constitutive equations, visit the Constitutive Background page.

Other Forms

The four state variables (**S**, **T**, **D**, and **E**) can be rearranged to give an additional 3 forms for a piezoelectric constitutive equation. Instead of the coupling matrix **d**, they contain the coupling matrices **e**, **g**, or **q**. It is possible to transform piezo constitutive data in one form to another form.

Why are these transformations important? A leading motivation is that vendors typically publish material data for **d** and **g**, whereas certain finite element codes require piezo data entered as **e**.

To view the 4 piezoelectric constitutive equations and their mutual transformations, visit the Constitutive Transform page.

All Constitutive Forms

The four possible forms for piezoelectric constitutive equations are shown below. The names for each of the forms is arbitrary; they were taken from the two dependent variables on the left-hand-side of each equation. Note that the Voltage and Electric Field variables are related via a gradient.

Strain-Charge Form:

$$\mathbf{S} = \mathbf{s}_E \cdot \mathbf{T} + \mathbf{d}^t \cdot \mathbf{E}$$
$$\mathbf{D} = \mathbf{d} \cdot \mathbf{T} + \boldsymbol{\varepsilon}_T \cdot \mathbf{E}$$

Stress-Charge Form:

$$\mathbf{T} = \mathbf{c}_E \cdot \mathbf{S} - \mathbf{e}^t \cdot \mathbf{E}$$
$$\mathbf{D} = \mathbf{e} \cdot \mathbf{S} + \boldsymbol{\varepsilon}_S \cdot \mathbf{E}$$

Strain-Voltage Form:

$$\mathbf{S} = \mathbf{s}_D \cdot \mathbf{T} + \mathbf{g}^t \cdot \mathbf{D}$$
$$\mathbf{E} = -\mathbf{g} \cdot \mathbf{T} + \boldsymbol{\varepsilon}_T^{-1} \cdot \mathbf{D}$$

Stress-Voltage Form:

$$\mathbf{T} = \mathbf{c}_D \cdot \mathbf{S} - \mathbf{q}^t \cdot \mathbf{D}$$
$$\mathbf{E} = -\mathbf{q} \cdot \mathbf{S} + \boldsymbol{\varepsilon}_S^{-1} \cdot \mathbf{D}$$

Transformations

Matrix transformations for converting piezoelectric constitutive data from one form into another form are shown below. Only 4 out of the 6 possible combinations are shown.

Strain-Charge to Stress-Charge:

$$\mathbf{c}_E = \mathbf{s}_E^{-1}$$
$$\mathbf{e} = \mathbf{d} \cdot \mathbf{s}_E^{-1}$$
$$\boldsymbol{\varepsilon}_S = \boldsymbol{\varepsilon}_T - \mathbf{d} \cdot \mathbf{s}_E^{-1} \cdot \mathbf{d}^t$$

Strain-Charge to Strain-Voltage:

$$\mathbf{s}_D = \mathbf{s}_E - \mathbf{d}^t \cdot \boldsymbol{\varepsilon}_T^{-1} \cdot \mathbf{d}$$
$$\mathbf{g} = \boldsymbol{\varepsilon}_T^{-1} \cdot \mathbf{d}$$

Stress-Charge to Stress-Voltage:

$$\mathbf{c}_D = \mathbf{c}_E + \mathbf{e}^t \cdot \boldsymbol{\varepsilon}_S^{-1} \cdot \mathbf{e}$$
$$\mathbf{q} = \boldsymbol{\varepsilon}_S^{-1} \cdot \mathbf{e}$$

Strain-Voltage to Stress-Voltage:

$$\mathbf{c}_D = \mathbf{s}_D^{-1}$$
$$\mathbf{q} = \mathbf{g} \cdot \mathbf{s}_D^{-1}$$
$$\boldsymbol{\varepsilon}_S^{-1} = \boldsymbol{\varepsilon}_T^{-1} + \mathbf{g} \cdot \mathbf{s}_D^{-1} \cdot \mathbf{g}^t$$

See the Symbol Definition page for the units and dimensions of the constitutive matrices shown here.

See Subscript Definitions for an explanation to the subscripts on the constitutive matrices.

Piezo Symbol Definitions

Following is a description of all matrix variables used in the piezoelectric constitutive equations.

Symbol	Object Type	Size	Units	Meaning
T	vector	6 x 1	$\frac{N}{m^2}$	stress components (e.g. σ_1)
S	vector	6 x 1	$\frac{m}{m}$	strain components (e.g. ε_3)
E	vector	3 x 1	$\frac{N}{C}$	electric field components
D	vector	3 x 1	$\frac{C}{m^2}$	electric charge density displacement components
s	matrix	6 x 6	$\frac{m^2}{N}$	compliance coefficients
c	matrix	6 x 6	$\frac{N}{m^2}$	stiffness coefficients
ε	matrix	3 x 3	$\frac{F}{m}$	electric permittivity
d	matrix	3 x 6	$\frac{C}{N}$	piezoelectric coupling coefficients for Strain-Charge form
e	matrix	3 x 6	$\frac{C}{m^2}$	piezoelectric coupling coefficients for Stress-Charge form
g	matrix	3 x 6	$\frac{m^2}{C}$	piezoelectric coupling coefficients for Strain-Voltage form
q	matrix	3 x 6	$\frac{N}{C}$	piezoelectric coupling coefficients for Stress-Voltage form

Lead Zirconate Titanate (PZT-2)

Crystal symmetry class	Uniaxial
Density	7600 kg/m^3

Constitutive data is presented in the Strain-Charge format.

Compliance

$$\mathbf{s_E} = \begin{bmatrix} 11.6 & -3.33 & -4.97 & 0 & 0 & 0 \\ -3.33 & 11.6 & -4.97 & 0 & 0 & 0 \\ -4.97 & -4.97 & 14.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 45 & 0 & 0 \\ 0 & 0 & 0 & 0 & 45 & 0 \\ 0 & 0 & 0 & 0 & 0 & 29.9 \end{bmatrix} *10^{-12}\ \frac{m^2}{N}$$

Piezoelectric Coupling

$$\mathbf{d} = \begin{bmatrix} 0 & 0 & 0 & 0 & 440 & 0 \\ 0 & 0 & 0 & 440 & 0 & 0 \\ -60 & -60 & 152 & 0 & 0 & 0 \end{bmatrix} *10^{-12}\ \frac{C}{N}$$

Relative Permittivity

$$\frac{\boldsymbol{\varepsilon}_\mathbf{T}}{\varepsilon_0} = \begin{bmatrix} 990 & 0 & 0 \\ 0 & 990 & 0 \\ 0 & 0 & 450 \end{bmatrix}, \quad \varepsilon_0 = 8.854 *10^{-12}\ \frac{F}{m}$$

Lead Zirconate Titanate (PZT-4)

Crystal symmetry class	Uniaxial
Density	7500 kg/m^3

Constitutive data is presented in the Strain-Charge format.

Compliance

$$\mathbf{s_E} = \begin{bmatrix} 12.3 & -4.05 & -5.31 & 0 & 0 & 0 \\ -4.05 & 12.3 & -5.31 & 0 & 0 & 0 \\ -5.31 & -5.31 & 15.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 39 & 0 & 0 \\ 0 & 0 & 0 & 0 & 39 & 0 \\ 0 & 0 & 0 & 0 & 0 & 32.7 \end{bmatrix} *10^{-12}\ \frac{m^2}{N}$$

Piezoelectric Coupling

$$\mathbf{d} = \begin{bmatrix} 0 & 0 & 0 & 0 & 496 & 0 \\ 0 & 0 & 0 & 496 & 0 & 0 \\ -123 & -123 & 289 & 0 & 0 & 0 \end{bmatrix} *10^{-12}\ \frac{C}{N}$$

Relative Permittivity

$$\frac{\boldsymbol{\varepsilon}_\mathbf{T}}{\varepsilon_0} = \begin{bmatrix} 1475 & 0 & 0 \\ 0 & 1475 & 0 \\ 0 & 0 & 1300 \end{bmatrix}, \quad \varepsilon_0 = 8.854 *10^{-12}\ \frac{F}{m}$$

Lead Zirconate Titanate (PZT-4D)

Crystal symmetry class	Uniaxial
Density	7600 kg/m^3

Constitutive data is presented in the Strain-Charge format.

Compliance

$$\mathbf{s}_E = \begin{matrix} 13.3 & -4.76 & -6.2 & 0 & 0 & 0 \\ -4.76 & 13.3 & -6.2 & 0 & 0 & 0 \\ -6.2 & -6.2 & 16.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 42 & 0 & 0 \\ 0 & 0 & 0 & 0 & 42 & 0 \\ 0 & 0 & 0 & 0 & 0 & 36.1 \end{matrix} \quad *10^{-12} \ \frac{m^2}{N}$$

Piezoelectric Coupling

$$\mathbf{d} = \begin{matrix} 0 & 0 & 0 & 0 & 550 & 0 \\ 0 & 0 & 0 & 550 & 0 & 0 \\ -135 & -135 & 315 & 0 & 0 & 0 \end{matrix} \quad *10^{-12} \ \frac{C}{N}$$

Relative Permittivity

$$\frac{\varepsilon_T}{\varepsilon_0} = \begin{matrix} 1610 & 0 & 0 \\ 0 & 1610 & 0 \\ 0 & 0 & 1450 \end{matrix} \quad , \quad \varepsilon_0 = 8.854 \ *10^{-12} \ \frac{F}{m}$$

Lead Zirconate Titanate (PZT-5A)

Crystal symmetry class	Uniaxial
Density	7750 kg/m^3

Constitutive data is presented in the Strain-Charge format.

Compliance

$$\mathbf{s}_E = \begin{bmatrix} 16.4 & -5.74 & -7.22 & 0 & 0 & 0 \\ -5.74 & 16.4 & -7.22 & 0 & 0 & 0 \\ -7.22 & -7.22 & 18.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 47.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 47.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 44.3 \end{bmatrix} *10^{-12}\ \frac{m^2}{N}$$

Piezoelectric Coupling

$$\mathbf{d} = \begin{bmatrix} 0 & 0 & 0 & 0 & 584 & 0 \\ 0 & 0 & 0 & 584 & 0 & 0 \\ -171 & -171 & 374 & 0 & 0 & 0 \end{bmatrix} *10^{-12}\ \frac{C}{N}$$

Relative Permittivity

$$\frac{\boldsymbol{\varepsilon}_T}{\varepsilon_0} = \begin{bmatrix} 1730 & 0 & 0 \\ 0 & 1730 & 0 \\ 0 & 0 & 1700 \end{bmatrix}, \quad \varepsilon_0 = 8.854 *10^{-12}\ \frac{F}{m}$$

Lead Zirconate Titanate (PZT-5J)

Crystal symmetry class	Uniaxial
Density	7400 kg/m^3

Constitutive data is presented in the Strain-Charge format.

Compliance

$$\mathbf{s}_E = \begin{matrix} 16.2 & -4.54 & -5.9 & 0 & 0 & 0 \\ -4.54 & 16.2 & -5.9 & 0 & 0 & 0 \\ -5.9 & -5.9 & 22.7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 47 & 0 & 0 \\ 0 & 0 & 0 & 0 & 47 & 0 \\ 0 & 0 & 0 & 0 & 0 & 41.5 \end{matrix} \quad {*10^{-12}} \ \frac{m^2}{N}$$

Piezoelectric Coupling

$$\mathbf{d} = \begin{matrix} 0 & 0 & 0 & 0 & 670 & 0 \\ 0 & 0 & 0 & 670 & 0 & 0 \\ -220 & -220 & 500 & 0 & 0 & 0 \end{matrix} \quad {*10^{-12}} \ \frac{C}{N}$$

Relative Permittivity

$$\frac{\boldsymbol{\varepsilon}_T}{\varepsilon_0} = \begin{matrix} 2720 & 0 & 0 \\ 0 & 2720 & 0 \\ 0 & 0 & 2600 \end{matrix} \quad , \quad \varepsilon_0 = 8.854 \ {*10^{-12}} \ \frac{F}{m}$$

Lead Zirconate Titanate (PZT-7A)

Crystal symmetry class	Uniaxial
Density	7700 kg/m^3

Constitutive data is presented in the Strain-Charge format.

Compliance

$$\mathbf{s_E} = \begin{matrix} 10.7 & -3.58 & -4.6 & 0 & 0 & 0 \\ -3.58 & 10.7 & -4.6 & 0 & 0 & 0 \\ -4.6 & -4.6 & 13.9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 34 & 0 & 0 \\ 0 & 0 & 0 & 0 & 34 & 0 \\ 0 & 0 & 0 & 0 & 0 & 28.6 \end{matrix} \quad {}^*10^{-12}\ \frac{m^2}{N}$$

Piezoelectric Coupling

$$\mathbf{d} = \begin{matrix} 0 & 0 & 0 & 0 & 360 & 0 \\ 0 & 0 & 0 & 360 & 0 & 0 \\ -60 & -60 & 153 & 0 & 0 & 0 \end{matrix} \quad {}^*10^{-12}\ \frac{C}{N}$$

Relative Permittivity

$$\frac{\boldsymbol{\varepsilon}_{\mathbf{T}}}{\varepsilon_0} = \begin{matrix} 930 & 0 & 0 \\ 0 & 930 & 0 \\ 0 & 0 & 425 \end{matrix} \quad , \quad \varepsilon_0 = 8.854\ {}^*10^{-12}\ \frac{F}{m}$$

Lead Zirconate Titanate (PZT-8)

Crystal symmetry class	Uniaxial
Density	7600 kg/m^3

Constitutive data is presented in the Strain-Charge format.

Compliance

$$\mathbf{s_E} = \begin{bmatrix} 11.5 & -3.7 & -4.8 & 0 & 0 & 0 \\ -3.7 & 11.5 & -4.8 & 0 & 0 & 0 \\ -4.8 & -4.8 & 13.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 31.9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 31.9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 30.4 \end{bmatrix} *10^{-12}\ \frac{m^2}{N}$$

Piezoelectric Coupling

$$\mathbf{d} = \begin{bmatrix} 0 & 0 & 0 & 0 & 330 & 0 \\ 0 & 0 & 0 & 330 & 0 & 0 \\ -0.97 & -0.97 & 225 & 0 & 0 & 0 \end{bmatrix} *10^{-12}\ \frac{C}{N}$$

Relative Permittivity

$$\frac{\boldsymbol{\varepsilon_T}}{\varepsilon_0} = \begin{bmatrix} 1290 & 0 & 0 \\ 0 & 1290 & 0 \\ 0 & 0 & 1000 \end{bmatrix}, \quad \varepsilon_0 = 8.854 *10^{-12}\ \frac{F}{m}$$

General Matrix Structure

The entries in the constitutive matrices for piezoelectric materials are subscript-labeled as follows:

$$\mathbf{s_E} = \begin{matrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ & & s_{33} & s_{34} & s_{35} & s_{36} \\ & & & s_{44} & s_{45} & s_{46} \\ & & & & s_{55} & s_{56} \\ & & & & & s_{66} \end{matrix}$$

Compliance
Matrix is symmetric.

$$\mathbf{d} = \begin{matrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{matrix}$$

Piezoelectric Coupling

$$\boldsymbol{\varepsilon_T} = \begin{matrix} \varepsilon_{11} & 0 & 0 \\ & \varepsilon_{22} & 0 \\ & & \varepsilon_{33} \end{matrix}$$

Permittivity
Matrix is symmetric, and diagonal for most materials.

Subscript Ordering

Constitutive data is presented in a 3-axis cartesian coordinate system, denoted by {*x*, *y*, *z*}.

The ordering of the 6 stress (and 6 strain) variables follows the convention used by crystallographers. This ordering determines the layout of the compliance matrix **s**, and determines the column ordering in the coupling matrix **d**.

The first 3 entries are the direct stresses along the *x*, *y*, and *z* axes, respectively. The final 3 entries are the shear stresses around the *x*, *y*, and *z* axes, respectively.

The electric field (and charge displacement) variable ordering is straightforward. The 3 entries correspond to the electric field component along the *x*, *y*, and *z* axes, respectively. This ordering determines the layout of the permittivity matrix ε, and determines the row ordering in the coupling matrix **d**.

For polarized piezoelectric materials (e.g. PZT), the poling direction is assumed to lie along the *z* axis.

Causion! Some finite element codes use a different ordering scheme for their stress and strain variables. Such a scheme would require the data in the **s** and **d** matrices to be re-arranged accordingly.

Watch out for this!