$n := 2$

$m := 0, 1 .. (2n) - 1$

$$F(m) := \left[\sum_{n=0}^{n-1} \left[\alpha_n \left(\beta_n \right)^m \right] \right]$$

$$F(m) \rightarrow \begin{bmatrix} \alpha_0 + \alpha_1 \\ \alpha_0 \cdot \beta_0 + \alpha_1 \cdot \beta_1 \\ \alpha_0 \cdot (\beta_0)^2 + \alpha_1 \cdot (\beta_1)^2 \\ \alpha_0 \cdot (\beta_0)^3 + \alpha_1 \cdot (\beta_1)^3 \end{bmatrix}$$

$$S(m) := \left(\int_{-1}^{1} x^m \, dx \right)$$

$$S(m) \rightarrow \begin{pmatrix} 2 \\ 0 \\ \frac{2}{3} \\ 0 \end{pmatrix}$$

$\alpha := 1 \qquad \beta := 1$

Given

$S(m) = F(m) \qquad S(m) = F(m) \rightarrow \begin{bmatrix} 2 = \alpha_0 + \alpha_1 \\ 0 = \alpha_0 \cdot \beta_0 + \alpha_1 \cdot \beta_1 \\ \frac{2}{3} = \alpha_0 \cdot (\beta_0)^2 + \alpha_1 \cdot (\beta_1)^2 \\ 0 = \alpha_0 \cdot (\beta_0)^3 + \alpha_1 \cdot (\beta_1)^3 \end{bmatrix}$

$\text{Find}(\alpha, \beta)$

$\alpha = 1 \qquad \beta = 1$