14.18. Reduced-Order Modeling for State-Space Matrices Export

The n second order modal equations (*Equation 14–118*) are transformed into 2n first order equations, where n is input as *NMODE* on the SPMWRITE command, using the following coordinate transformation:

$$\{z\} = \begin{Bmatrix} y \\ \dot{y} \end{Bmatrix} \qquad \textbf{(14–280)}$$

The equation becomes:

$$\{\dot{z}\} = [A]\{z\} + [B]\{F\} \qquad \textbf{(14–281)}$$

[A] is a (2n x 2n) state-space matrix defined by:

$$[A] = \begin{bmatrix} 0 & I \\ \Gamma_1 & \Gamma_2 \end{bmatrix} \qquad \textbf{(14–282)}$$

$$[\Gamma_1] = \begin{bmatrix} -\omega_1^2 & 0 & 0 & & \\ 0 & \dots & & & \\ 0 & & -\omega_j^2 & & 0 \\ & & & \dots & 0 \\ & & 0 & 0 & -\omega_n^2 \end{bmatrix}$$

$$[\Gamma_2] = \begin{bmatrix} -2\zeta_1\omega_1 & 0 & 0 & & \\ 0 & \dots & & & \\ 0 & & -2\zeta_j\omega_j & & 0 \\ & & & \dots & 0 \\ & & 0 & 0 & -2\zeta_n\omega_n \end{bmatrix}$$

Where ω_j is the frequency of mode j, ξ_j is the effective modal damping of mode j (see *Modal Damping*), and {F} is the vector of input forces:

$$\{F\} = \begin{Bmatrix} F_1(t) \\ \dots \\ F_{ninput}(t) \end{Bmatrix} \qquad \textbf{(14–283)}$$

Where ninput is the number of scalar input forces derived from *Inputs* on the SPMWRITE command.

[B] is a (2n x ninput) state-space matrix defined by:

$$[B] = \begin{bmatrix} 0 \\ \Gamma_3 \end{bmatrix} \quad (14\text{–}284)$$

With

$$[\Gamma_3] = [\Phi]^T [F_u] \quad (14\text{–}285)$$

Where [Φ] is the matrix of eigenvectors and [F_u] is a unit force matrix with size (ndof x ninput). It has 1 at the degrees of freedom where input forces are active and 0 elsewhere.

Now that the states {z} have been expressed as a function of the input loads, the equation for the degrees of freedom observed (outputs w) is written as:

$$\begin{Bmatrix} w \\ \dot{w} \\ \ddot{w} \end{Bmatrix} = [C]\{z\} + [D]\{F\} \quad (14\text{–}286)$$

[C] is a (3*noutput x 2*n) state-space matrix, where noutput is derived from *Outputs* on the SPMWRITE command, and is defined by:

$$[C] = \begin{bmatrix} \Gamma_4 & 0 \\ 0 & \Gamma_4 \\ \Gamma_4\Gamma_1 & \Gamma_4\Gamma_2 \end{bmatrix} \quad (14\text{–}287)$$

with

$$[\Gamma_4] = [U_u][\Phi] \quad (14\text{–}288)$$

[U_u] is a unit displacement matrix with size (noutput x ndof). It has 1 on degrees of freedom where output is requested and 0 elsewhere.

[D] is a (3*noutput x ninput) state-space matrix defined by:

$$[D] = \begin{bmatrix} 0 \\ 0 \\ \Gamma_4\Gamma_3 \end{bmatrix} \quad (14\text{–}289)$$

$\dot{w}$ **and** $\ddot{w}$ **are included only if VelAccKey = ON on the SPMWRITE command, otherwise the last two rows of [C] are not written and [D] is zero so it is not written.**