

14.3 Derivations

The following table provides additional definitions for the selected result derivations. These include tensor to vector, tensor to scalar, and vector to scalar resolutions.

Transform Type	Derivation Method	Description
Scalar to Scalar Vector to Vector Tensor to Tensor	None	No transformation is used if the result data type matches the plot tool's data type.
Vector to Scalar	Magnitude	Vector magnitude.
	X Component	1st vector component.
	Y Component	2nd vector component.
	Z Component	3rd vector component.
Tensor to Scalar	XX Component	XX tensor component.
	YY Component	YY tensor component.
	ZZ Component	ZZ tensor component.
	XY Component	XY tensor component.
	YZ Component	YZ tensor component.
	ZX Component	ZX tensor component.
	Min Principal	Calculated minimum principal magnitude.
	Mid Principal	Calculated middle principal magnitude.
	Max Principal	Calculated maximum principal magnitude.
	1st Invariant	Calculated 1st invariant
	2nd Invariant	Calculated 2nd invariant
	3rd Invariant	Calculated 3rd invariant
	Hydrostatic	Calculated mean of the three normal tensor components.
	von Mises	Calculated effective stress using von Mises criterion.
	Tresca	Calculated Tresca shear stress.
Max Shear	Calculated maximum shear magnitude.	
Octahedral	Calculated Octahedral shear stress.	
Tensor to Vector	Min Principal	Calculated minimum principal vector.
	Mid Principal	Calculated middle principal vector.
	Max Principal	Calculated maximum principal vector.

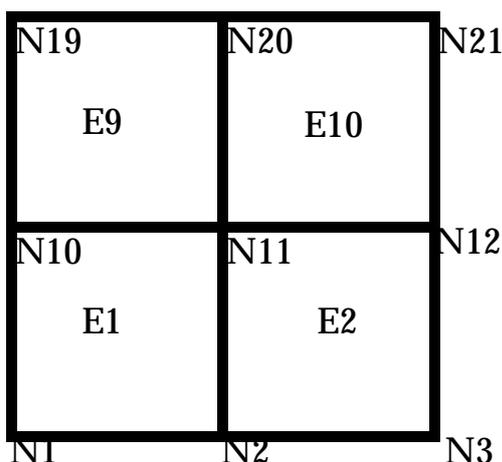
Below are the equations and examples of the derivation methods:

Important: These equations for calculating invariants are not recommended for complex results since phase is not taken into account.

von Mises Stress. von Mises stress is calculated from the following equation:

$$\sigma' = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2}{2} + 3(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}$$

Example: The elements shown below have the following stress contributions:



Elem. ID	Node ID	σ_x	σ_y	σ_z	τ_{xy}	τ_{yz}	τ_{zx}
1	1	46.2	13.01	0.00	5.13	0.00	0.00
	2	93.39	25.33	0.00	17.45	0.00	0.00
	11	68.37	12.16	0.00	-19.73	0.00	0.00
	10	44.32	10.40	0.00	-1.01	0.00	0.00
2	2	93.39	25.33	0.00	17.45	0.00	0.00
	3	88.67	24.41	0.00	23.95	0.00	0.00
	12	57.42	5.44	0.00	-34.02	0.00	0.00
	11	59.37	10.16	0.00	-20.73	0.00	0.00
9	10	44.32	10.40	0.00	-1.01	0.00	0.00
	11	67.37	11.16	0.00	-18.73	0.00	0.00
	20	4.72	8.15	0.00	-15.28	0.00	0.00
	19	17.99	7.68	0.00	-4.61	0.00	0.00

Elem. ID	Node ID	σ_x	σ_y	σ_z	τ_{xy}	τ_{yz}	τ_{zx}
10	11	100.37	14.16	0.00	-30.73	0.00	0.00
	12	57.42	5.44	0.00	-34.02	0.00	0.00
	21	-5.63	5.72	0.00	-22.03	0.00	0.00
	20	4.72	8.15	0.00	-15.28	0.00	0.00

The von Mises stress calculated at node 11 when nodal averaging is done first due to the contribution from each element is 78.96. When the von Mises derivation is done first and then averaging at the nodes takes place, the calculated von Mises stress is 79.02. Thus a difference can arise depending on whether the averaging is done first or the derivation. This can be true for all derived results.

Node 11	σ_x	σ_y	σ_z	τ_{xy}	τ_{yz}	τ_{zx}	von Mises Stress
E1	68.37	12.16	0.00	-19.73	0.0	0.0	71.82
E2	59.37	10.16	0.00	-20.73	0.00	0.00	65.68
E9	67.37	11.16	0.00	-18.73	0.00	0.00	70.45
E10	100.37	14.16	0.00	-30.73	0.00	0.00	108.10
Average	73.87	11.91	0.00	-22.48	0.00	0.00	79.02
Average then Derive							78.96
Derive then Average							79.02

Important: It must be noted also that for von Mises and other derived results, the calculations are generally valid only for stresses. Although these operations can be performed for any valid tensor or vector data stored in the database, quantities such as tensor strains are not appropriate for von Mises calculations. To calculate a true von Mises strain the strain tensor must be converted to engineering strains by multiplying the shear components by a factor of two.

Octahedral Shear Stress. Octahedral shear stress is calculated from the following equation:

$$\tau_{oct} = \frac{\sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}}{3}$$

From the von Mises example above the octahedral shear stress is:

Octahedral Shear Stress	Node 11
Average/Derive	37.22
Derive/Average	37.25

Hydrostatic Stress. Hydrostatic stress is calculated from the following equation:

$$\sigma = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

From the von Mises example above the hydrostatic stress is:

Hydrostatic Stress	Node 11
Average/Derive	28.59
Derive/Average	28.59

Invariant Stresses. 1st, 2nd, and 3rd invariant stresses are calculated from the following equations:

$$\sigma_{1st} = (\sigma_x + \sigma_y + \sigma_z)$$

$$\sigma_{2nd} = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_z \sigma_x - (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)$$

$$\sigma_{3rd} = \sigma_x(\sigma_y \sigma_z - \tau_{yz}^2) + \tau_{xy}(\tau_{xy} \sigma_z - \tau_{yz} \tau_{zx}) + \tau_{zx}(\tau_{xy} \tau_{yz} - \sigma_x \tau_{zx})$$

From the von Mises example above the invariant stresses are:

Invariant Stresses (Node 11)	1st Invariant	2nd Invariant	3rd Invariant
Average/Derive	85.78	374.44	0.00
Derive/Average	85.78	373.38	0.00

Principal Stresses. Principal stresses are calculated from either a Mohr's circle method for 2D tensors ($\sigma_z = \tau_{yz} = \tau_{zx} = 0$) or from a 3x3 Jacobian Rotation Eigenvector extraction method for a 3D tensors. The User Interface allows for either a tensor-dependent derivation, or a 2D calculation. The tensor-dependent calculation will choose either a 2D or 3D calculation depending on values of each tensor encountered. A 2D calculation will be used when the ZZ, YZ and ZX are exactly zero (which is the case when the analysis code does not calculate these values), otherwise the full 3D tensor will be considered. Both the magnitudes of the principals and their direction cosines are calculated from these routines.

The magnitudes of the two principal stresses from the 2D Mohr's circle method are calculated according the following equations:

$$\sigma_{major} = \sigma_{ave} + \sqrt{(\sigma_x - \sigma_{ave})^2 + \tau_{xy}^2}$$

$$\sigma_{minor} = \left(\sigma_{ave} - \sqrt{(\sigma_x - \sigma_{ave})^2 + \tau_{xy}^2} \right)$$

where:

$$\sigma_{ave} = \frac{(\sigma_x + \sigma_y)}{2}$$

The direction cosines for the 2D Mohr's circle method are calculated by assembling the following 3x3 transformation matrix:

$$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{where } \theta = \frac{\text{atan}\left(\frac{\tau_{xy}}{\sigma_x - \sigma_{ave}}\right)}{2}$$

From the von Mises example above the principal stresses are:

Principal Stresses (Node 11)	Maximum	Minimum
Average/Derive	81.17	4.61
Derive/Average	81.20	4.58

Also the principal stress determinant is the product of the three principals and the major, minor, and intermediate principal deviatoric stresses are calculated from:

$$\sigma_{(maj, dev)} = \sigma_{major} - \frac{(\sigma_{major} + \sigma_{inter} + \sigma_{minor})}{3}$$

$$\sigma_{(min, dev)} = \sigma_{minor} - \frac{(\sigma_{major} + \sigma_{inter} + \sigma_{minor})}{3}$$

$$\sigma_{(int, dev)} = \sigma_{inter} - \frac{(\sigma_{major} + \sigma_{inter} + \sigma_{minor})}{3}$$

Tresca Shear Stress. Tresca shear stress is calculated from the following equation:

$$\tau = (\sigma_{major} - \sigma_{minor})$$

where σ_{major} and σ_{minor} are calculated as mentioned under Principal stress derivations above.

From the von Mises example above the Tresca shear stress is:

Tresca Shear Stress	Node 11
Average/Derive	76.55
Derive/Average	76.61

Maximum Shear Stress. Maximum shear stress is calculated from the following equation

$$\tau = \frac{(\sigma_{major} - \sigma_{minor})}{2}$$

where σ_{major} and σ_{minor} are calculated as mentioned under Principal stress derivations above.

From the von Mises example above the Tresca shear stress is:

Tresca Shear Stress	Node 11
Average/Derive	76.55
Derive/Average	76.61

Magnitude. Magnitude (vector length) is calculated from the components with the standard formula:

$$magnitude = \sqrt{x^2 + y^2 + z^2}$$

14.4 Averaging

For Fringe and other plots and reports that must display or report values at nodes from elemental data regardless of where the element results are computed, must be converted to results at element nodes. The interpolation functions are then used (e.g., by the graphics module for fringe plot and other operations) to compute the results at any point within the element. The interpolation functions may or may not be the shape functions that were used by the analysis program to compute the element results.

As a rule, each element sharing a common node has its own result values. To compute results for continuous fringe plots, these values need to be averaged and distributed to the sharing elements. The options for the averaging process are described below:

No Averaging	Each element retains its value at the element nodes. Or in other words, each element is its own averaging domain. This selection from the Averaging Domain pull down is called None. The fringe plot will have jumps (not continuous regions) at element boundaries.
Averaging Based on All Entities	All elements will contribute to the sum and will receive the averaged result regardless of whether only certain entities have been selected for the display of the fringe plot. All surrounding elements will contribute to the averaging process.
Averaging Based on Target Entities	Only the elements defined as the target entities will contribute to the sum and will receive the averaged result. Surrounding elements that are not part of the target entities will not contribute to the averaging process.
Averaging Based on Materials	Elements with the same material IDs will contribute to the sum and will receive the averaged result. The fringe plot will have jumps at material boundaries.
Averaging Based on Properties	Elements with the same property IDs will contribute to the sum and will receive the averaged result. The fringe plot will have jumps at property boundaries.
Averaging Based on Element Types	Elements of the same type will contribute to the sum and will receive the averaged result. The fringe plot will have jumps at element type boundaries.
Difference	The minimum and maximum results from the elements sharing a common node are computed. The difference is determined as the delta between the maximum and minimum contributor to each node. The fringe plot of this max difference indicates the quality of the mesh and the location where this mesh needs to be refined by comparing its values with the actual values of the results. Nodal results will have zero max-difference.
Sum	The sum of all contributing nodes will be displayed. This step skips the averaging.

Below are some examples of the averaging techniques. The model in **Figure 14-1** is used for illustration purposes. It consists of 8 QUAD4 elements and 4 TRI3 elements with a total of 17 nodes.

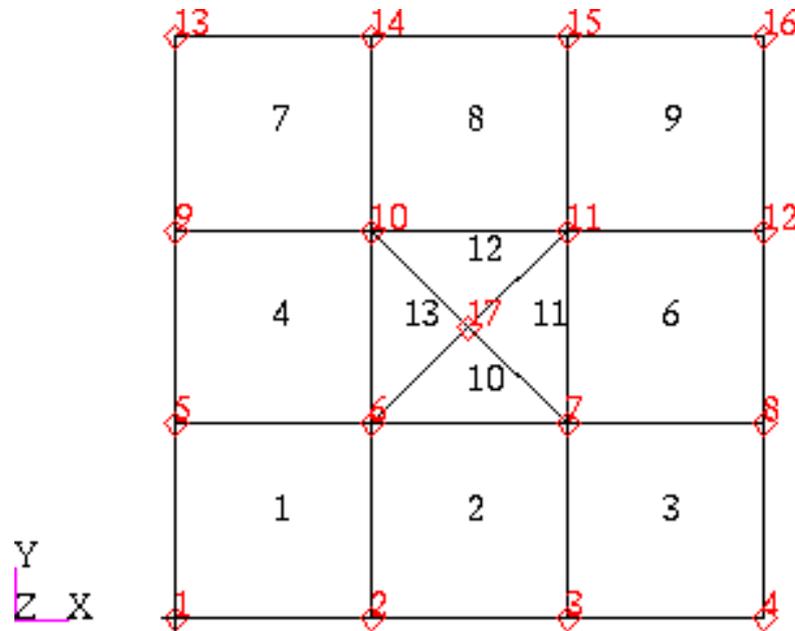


Figure 14-1 Square Plate Model to Illustrate Averaging Techniques.

The above model is also broken up into various material and property sets as such:

Prop1	Mat1	Elem 1:3
Prop2	Mat2	Elem 6 8:9
Prop3	Mat3	Elem 4 7
Prop4	Mat1	Elem 10:13
Target1		Elem 1:3 6 10:11
Target2		Elem 4 7:9 12:13

Element Centroidal Results. The first illustration is the simple case of results at element centroids. [Table 14-1](#) below lists some scalar values of strain energy at each element centroid. The table is listed by node number with each element and corresponding strain energy value for all contributing elements associated with the particular nodes. The averaging domain columns on the right then list the averaged values for each node based on the averaging domain. Columns with more than one value per node indicate a boundary of the averaging domain and will therefore cause a plot discontinuity across boundaries. See [Figure 14-2](#) for visual effects of averaging domains.

Table 14-1 Averaging at Nodes from Element Centroidal Results

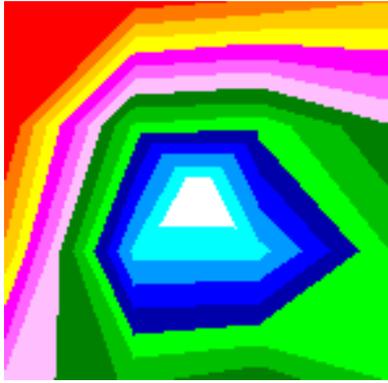
Node	Element	Strain Energy	Averaging Domain							
			All	Property	Material	None	Type	Target		
1	1	3.01	3.01	3.01	3.01	3.01	3.01	3.01		
2	1	3.01	3.89	3.89	3.89	3.01	3.89	3.89		
	2	4.78				4.78				
3	2	4.78	3.97	3.97	3.97	4.78	3.97	3.97		
	3	3.16				3.16				
4	3	3.16	3.16	3.16	3.16	3.16	3.16	3.16		
5	1	3.01	8.04	3.01		3.01	8.04	3.01		
	4	13.06		13.06	13.06	13.06				
6	1	3.01	4.24	3.89	2.04	3.01	6.95	2.63		
	2	4.78				4.78				
	4	13.06		13.06	13.06	13.06				
	10	0.10		0.19	2.04	0.10			0.19	2.63
	13	0.27				0.27				6.67
7	2	4.78	2.09	3.97	2.04	4.78	3.42	2.09		
	3	3.16				3.16				
	6	2.31		2.31	2.04	2.31				
	10	0.10		0.11	2.04	0.10			0.11	
	11	0.11				0.11				
8	3	3.16	2.74	3.16	3.16	3.16	2.74	2.74		
	6	2.31		2.31	2.31	2.31				
9	4	13.06	12.10	12.10	12.10	13.06	12.10	12.10		
	7	11.13				11.13				

Table 14-1 Averaging at Nodes from Element Centroidal Results (continued)

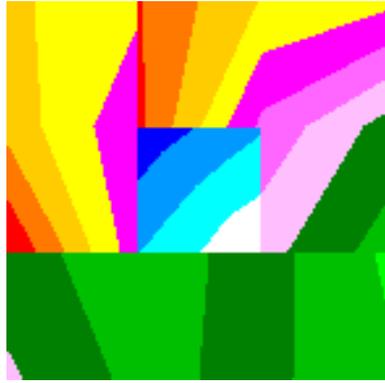
Node	Element	Strain Energy	Averaging Domain						
			All	Property	Material	None	Type	Target	
10	4	13.06	5.95	12.01	12.01	13.06	9.74	5.95	
	7	11.13				11.13			
	8	5.02		5.02	5.02	5.02			
	12	0.27		0.27	0.27	0.27			
	13	0.27				0.27			
11	6	2.31	2.11	3.38	3.38	2.31	3.38	1.21	
	8	5.02				5.02		2.70	
	9	2.82				2.82			
	11	0.11		0.19	0.19	0.11		0.19	1.21
	12	0.27				0.27			
12	6	2.31	2.57	2.57	2.57	2.31	2.57	2.31	
	9	2.82				2.82		2.82	
13	7	11.13	11.13	11.13	11.13	11.13	11.13	11.13	
14	7	11.13	8.08	11.13	11.13	11.13	8.08	8.08	
	8	5.02		5.02	5.02	5.02			
15	8	5.02	3.92	3.92	3.92	5.02	3.92	3.92	
	9	2.82				2.82			
16	9	2.82	2.82	2.82	2.82	2.82	2.82	2.82	
17	10	0.10	0.19	0.19	0.19	0.10	0.19	0.10	
	11	0.11				0.11			
	12	0.27				0.27			0.27
	13	0.27				0.27			0.27

Table 14-2 Averaging at Nodes from Element Nodal Results (continued)

Element	Node	von Mises Stress	Averaging Domain					
			All	Property	Material	None	Type	Target
8	10	397215	310705	397215	397215	397215	350090	310705
	11	389998	265760	311763	311763	389998	311763	305499
	15	384259	346068	346068	346068	384259	346068	346068
	14	391346	361658	391346	391346	391346	361658	361658
9	11	275878	265760	311763	311763	275878	311763	305499
	12	297297	264210	264210	264210	297297	264210	297297
	16	331799	331799	331799	331799	331799	331799	331799
	15	307878	346068	346068	346068	307878	307878	346068
10	6	144769	238950	198700	240096	144769	198700	209085
	7	144769	213334	143829	199024	144769	143829	213334
	17	144769	197728	197728	197728	144769	197728	143829
11	7	142890	213334	143829	199024	142890	143829	213334
	11	142890	265760	196756	196756	142890	196756	206152
	17	142890	197728	197728	197728	142890	197728	143829
12	11	250623	265760	196756	196756	250623	196756	305499
	10	250623	310705	251626	251626	250623	251626	310705
	17	250623	197728	197728	197728	250623	197728	251627
13	10	252631	310705	251626	251626	252631	251626	310705
	6	252631	238950	198700	240096	252631	198700	283747
	17	252631	197728	197728	197728	252631	197728	251627



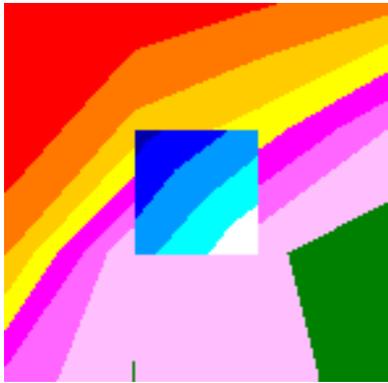
All Entities



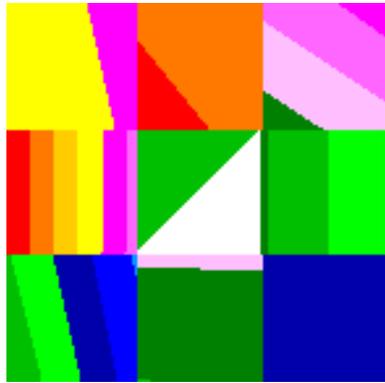
By Property



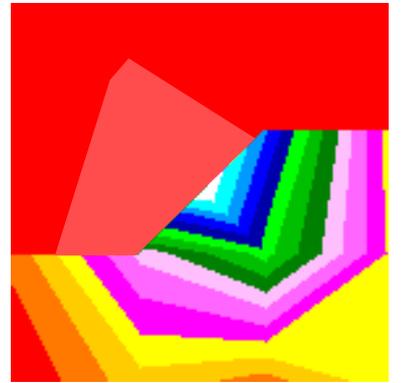
By Material



By Element Type



None



By Target Entity

Figure 14-2 Differences in Plots Due to Averaging Domains - Note Discontinuities.

14.5 Extrapolation

When element results are provided to MSC.Patran at quadrature points, it is necessary to extrapolate the results from the quadrature points to the nodes of the element and to the element centroid. Similarly, when results are provided at the element nodes or the centroid, it is necessary to interpolate/extrapolate the results to the centroid or nodes respectively.

MSC.Patran has three basic methods to perform this interpolation/extrapolation:

- By parametric mapping method.
- By solving a set of equations.
- By averaging.

The User Interface allows for four basic methods in which the user can control extrapolation methods. These are explained below and examples given.

Shape Function. If the arrangement of node/quadrature points corresponds to an element type in MSC.Patran, the shape functions are known, and a parametric mapping is used. This is the preferred method, and is the most accurate representation. The parametric mapping method involves mapping the output positions to an element topology that interpolation functions of that topology can be used to compute results at the nodes. As an example, if there are 27 results output at 27 quadrature points inside a hex/20, then these 27 quadrature points can be considered as 27 vertices of a hex/27 element. Results at hex/20 nodes are then computed by the interpolation function of the hex/27, even though these nodes are located outside the element formed by the 27 quadrature points. Once the nodal results of the hex/27 are available, results at the nodes of the hex/20 can be computed by interpolation. These results will be stored in a 20x27 matrix of coefficients. This method only works if there exists an element topology in the library that coincides with the output pattern after being parametrically mapped.

If the arrangement does not correspond to a MSC.Patran element type, a system of equations is constructed and solved for the unknown nodal and centroidal values. The equations are set up such that if the interpolation functions of the element topology are used with the unknown nodal values, they will generate a unit value at each quadrature point. This method only works if there exists an element topology in the library that has the same number of nodes as the number of quadrature points. If Shape Function is set in the User Interface, the shape functions or a set of equations will be used to extrapolate results as explained above. Only if these two methods fail will averaging take place.

Average. If both previous methods fail, results in the element are averaged and each node of the element will assume this averaged value. Or, alternatively, if the results are provided at nodes, the nodal values would be averaged and assigned to the centroid.

Averaging is also used at element boundaries. In these cases, when extrapolation from different elements yields different result values at the same node, the different results are averaged and assigned to the node.

For degenerate elements, the extrapolation is performed on the parent element topology, and the results at the duplicated nodes in the degenerate element are then averaged.

The User Interface allows for a forced average extrapolation method to be used. The following scenarios can exist.

- Nodal values to centroid
- Gauss values to nodes
- Centroidal values to nodes

Centroid. A forced extrapolation of the analysis results to the element's centroid can also be set in the User Interface which will be performed relative to where the results are initially located. Shown below are several different cases that can occur. Once each centroid value is established it is then used to render the results plot.

- Centroid values to element centroid
- Nodal values to element centroid
- Gauss values to element centroid

Min/Max. The Min/Max method searches each element's results and finds the minimum/maximum value contained within the element. The element then assumes a constant value (including its nodes). For example if the analysis result values are known at the element's Gauss points the minimum/maximum value is used as a constant value across the element. This method has no effect for results that already exist at the element centroid or the nodes.

Examples. Examples are given below for each extrapolation technique using a simple 4 node QUAD element with four interior Gauss points. The Gauss points are located in parametric space at +/- 0.5773502692 (as per theory). In p/q parametric space, where the extrapolation occurs, would look something like **Figure 14-3**.

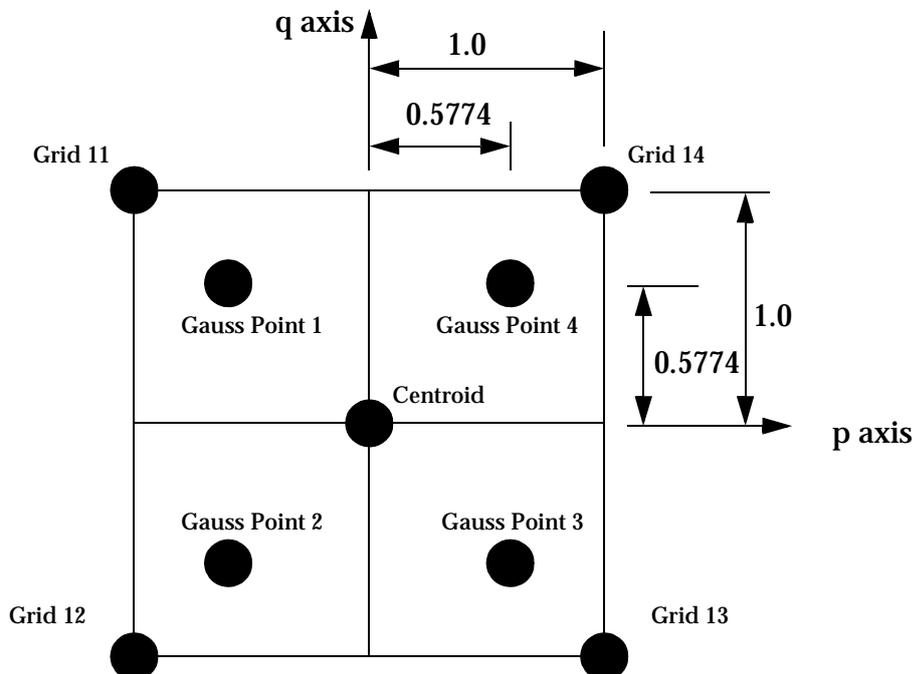


Figure 14-3 Example 4 Noded QUAD with Gauss Points.

The element will have a simple set of linear shape functions described by

$$N_1 = -(p - 1)(q + 1)$$

$$N_2 = (p - 1)(q - 1)$$

$$N_3 = -(p + 1)(q - 1)$$

$$N_4 = (p + 1)(q + 1)$$

Using these shape functions, the results at any point in the element would be found as

$$Result(p, q) = \sum N_i(p, q) \times Result_i$$

where i runs from 1 to 4 for the four Gauss or grid points.

Note that the shape functions vary by element type and element order. The function shown in these examples are not necessarily the functions used in any particular element formulation; they are to illustrate the extrapolation methods only.

Example 1 - Parametric Mapping (Gauss points to element nodes)

Gauss point results are as follows:

Gauss Point	Stress
1	10.
2	15.
3	20.
4	15.

The stress values at the Gauss points will be extrapolated to the grid locations. To do this, the Gauss points are assigned parametric locations of 1.0. The location of the grids will be at parametric locations of 1/0.5774 or about +/-1.7319 with respect to the Gauss points.

The stress at grid 14, located in parametric space at x/y coordinates of (1.7319, 1.7319) will be calculated as:

$$-\frac{1}{4}(1.7319 - 1)(1.7319 + 1) \times 10 + \frac{1}{4}(1.7319 - 1)(1.7319 - 1) \times 15 + -\frac{1}{4}(1.7319 + 1)(1.7319 - 1) \times 20 + \frac{1}{4}(1.7319 + 1)(1.7319 + 1) \times 15 = 15.00$$

The stresses at the rest of the grids would be as follows:

Grid	X Location	Y Location	Stress
11	-1.7319	1.7319	6.340499
12	-1.7319	-1.7319	15.00
13	1.7319	-1.7319	23.65950
14	1.7319	1.7319	15.00

Example 2 - Parametric Mapping (Gauss points to element centroid)

The stress at the Gauss points are the same as Example 1. The element centroid would be located in parametric space at (0,0), so interpolation to that point can be accomplished directly:

$$-\frac{1}{4}(0 - 1)(0 + 1) \times 10 + \frac{1}{4}(0 - 1)(0 - 1) \times 15 + -\frac{1}{4}(0 + 1)(0 - 1) \times 20 + \frac{1}{4}(0 + 1)(0 + 1) \times 15 = 15.00$$

Example 3 - Parametric Mapping (Nodal results to element centroid)

In this example the results at the grid points are provided to MSC.Patran. To make an element fill plot, the element centroidal value must be known. The stress values at the element grid points are:

Gauss Point	Stress
1	6.340499
2	15.00
3	23.65950
4	15.00

The value at the centroid is then calculated using the shape functions, just as in Example 2 above:

$$-\frac{1}{4}(0-1)(0+1) \times 10 + \frac{1}{4}(0-1)(0-1) \times 15 + -\frac{1}{4}(0+1)(0-1) \times 20 + \frac{1}{4}(0+1)(0+1) \times 15 = 15.00$$

Note that this gives the same results as in the previous example.

Example 4 - Averaging (Nodal results to element centroid)

The averaging technique simply computes the mathematical average of the nodal stresses and reports this as the centroidal value. So, the centroidal stress would be reported as:

$$(6.340499 + 15 + 23.65950 + 15) / 4 = 15.00$$

Example 5 - Averaging (Gauss points to element nodes)

In this case no suitable set of shape functions exists to carry out a proper interpolation. Therefore, the Gauss point stresses are averaged, and the average result distributed to all the grid points:

$$(10 + 15 + 20 + 15) / 4 = 15.00$$

The grid point stresses would be reported as:

Grid Point	Stress
11	15.00
12	15.00
13	15.00
14	15.00

Example 6 - Averaging (Centroidal values to element nodes)

In this case there is only one piece of stress data available, so no assumptions about the stress distribution can be made. Therefore, if the element centroid stress is reported as 15.00, the grid point stress will be reported as:

Grid Point	Stress
11	15.00
12	15.00
13	15.00
14	15.00

Example 7 - Averaging (Adjacent element contributions)

In this case the stresses in an adjacent element are included in the reporting of the grid point stress. If two elements have nodal stresses calculated from Gauss points by internal extrapolation as follows:

Element 1		Element 2	
Grid Point	Stress	Grid Point	Stress
11	6.340499	13	27.50
12	15.00	14	17.50
13	23.65950	15	10.00
14	15.00	16	9.50

The nodal stresses calculated by MSC.Patran would be:

Grid Point	Stress
11	6.340499
12	15.00
13	$25.5798 = [(23.65650 + 27.50) / 2]$
14	$16.25 = [(15.00 + 17.50) / 2]$
15	10.00
16	9.50