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On the lobe profile design in a cycloid reducer using instant velocity center

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Abstract

A cycloid speed reducer is one of the rotational speed regulation devices of the machinery. It has advantages of the higher reduction ratio, the higher accuracy, the easier adjustment of the transmission ratio and the smaller workspace than any other kinds of the reducer. This paper proposes a simple and exact approach for the lobe profile design of the cycloid plate gear, which is a main part of the cycloid reducer, by means of the principle of the instant velocity center in the general contact mechanism and the homogeneous coordinate transformation. It is considered the four types of the cycloid reducers in this study: the stationary ring gear type epicycloid reducer, the rotating ring gear type epicycloid reducer. Design examples for the four types of the cycloid reducers are presented to simulate the operation and to demonstrate the feasibility of this approach using a computer-aided program developed on C++ language. © 2005 Elsevier Ltd. All rights reserved.

Keywords: Cycloid reducer; Epicycloid plate gear; Hypocycloid plate gear; Instant velocity center; Homogeneous coordinate transformation

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1. Introduction

Speed reducers are used widely in various applications for speed and torque conversion purposes. Among them, a cycloid reducer has been used for decades owing to their smooth and high performance, high reliability, long service life, compactness, exceptional overload capacity, low to zero backlash through rolling tooth engagement in the contact mechanism, and other advantages. Therefore it makes an attractive candidate for limited space applications today.

A cycloid plate gear, which is a main part of the cycloid reducer, meshes in all teeth or lobes at any one time with the roller gear (or ring gear) consisted of several rollers on the circular pitch line. Generally, it is classified into four types of the cycloid drives by the lobe profile of the cycloid plate gear and the roller gear's motion: the stationary ring gear type epicycloid reducer, the rotating ring gear type epicycloid reducer, the stationary ring gear type hypocycloid reducer and the rotating ring gear type hypocycloid reducer.

For an example, the stationary ring gear type epicycloid reducer (see Fig. 1) basically has only three major moving parts: high speed input shaft with integrally mounted eccentric cam and roller bearing assembly corresponding to the distance of centers between roller gear and cycloidal plate gear, cycloidal plate gear, and slow speed output shaft assembly. As the eccentric cam rotates, it rolls the cycloid plate gears around the internal circumference of the stationary ring gear. The resulting action is similar to that of a wheel rolling around the inside of a ring. As the wheel (cycloidal plate) travels around the ring gear, the wheel itself turns slowly on its own axis in an opposite direction. That is, for each complete revolution of the high speed shaft the cycloidal plate gear turns one lobe pitch in the opposite direction. In general, there is one less cycloidal tooth around the plate gear than there are rollers in the stationary ring gear housing, which results in reduction ratios being numerically equal to the number of lobes on the plate gear. The reduced rotation of the plate gears is transmitted to the slow speed output shaft, not depicted in Fig. 1,

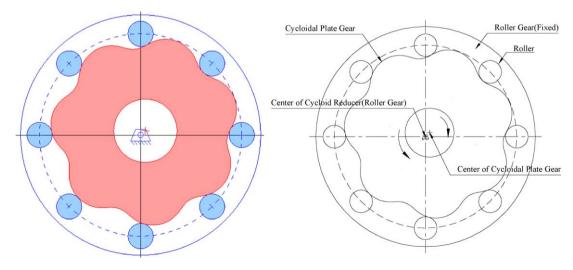


Fig. 1. Shape of a stationary ring gear type epicycloid reducer.

by means of drive pins and rollers which engage with holes located around the middle of each plate gear.

To the authors' best knowledge, little published information is available on analysis and design of the cycloid reducer. Botsiber and Kingston [1] introduced, with little analytical work, the theory of operation of the cycloid drive mechanism. Malhotra and Parameswaran [2] studied the effects of design parameters on forces for various elements of the cycloid speed reducer as well as the theoretical efficiency. Blanche and Yang [3] developed an analytical model of the cycloid drives with machining tolerances and investigated the effects of machining tolerances on backlash and torque ripple; and they [4] also presented a computer-aided analysis procedure to verify the performance of cycloid drives. Litvin and Feng [5] used differential geometry to generate the conjugate surfaces of cycloidal gearing. Recently, Yan and Lai [6] have presented a geometric design concept of a hypocycloidal reducer using the theory of conjugate surfaces. Most recently, Li et al. [7] have introduced a double crank ring-plate-type cycloid drive and presented its working principles, advantages and design issues.

In this paper, we propose a new approach for the exact geometric design of the cycloidal plate gears without interference in the cycloid drives using the principle of instant velocity center and the homogeneous coordinate transformation technique. It is considered the four types of the cycloid reducers in this study; the stationary ring gear type epicycloid reducer in Section 2, the rotating ring gear type epicycloid reducer in Section 3, the stationary ring gear type hypocycloid reducer. Based upon the proposed approach, a program for shape design automation has been developed with C++ language. Finally, design examples are presented to demonstrate the feasibility of this approach.

2. Stationary ring gear type epicycloid reducer

According to Kennedy's theorem [8–10], the three instant velocity centers shared by three rigid bodies in relative motion to one another (whether or not connected) all lie on the same straight line. Fig. 2 shows the construction necessary to find instant velocity centers. In Fig. 2 links 2 and 3 are in direct contact. All pin joints (IC_{12} , IC_{13}) are permanent instant centers. If the point of contact does not lie on the line of centers IC_{12} – IC_{13} , these tangential components will not be

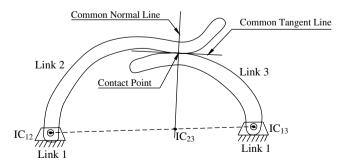


Fig. 2. Instant velocity centers of a contact mechanism.

equal, and sliding exists. Hence the only relative motion which links 2 and 3 can have at their point of contact is in the direction of the common tangent, and their center of relative rotation, instant velocity center IC_{23} , must then lie along the common normal. However, by Kennedy's theorem instant velocity center IC_{23} must lie along line IC_{12} – IC_{13} . Hence instant velocity center IC_{23} must lie along line IC_{12} – IC_{13} . Hence instant velocity center IC_{23} lies at the point of the intersection of the common normal and the line of centers IC_{12} – IC_{13} .

Fig. 3 is a schematic of a stationary ring gear type epicycloid reducer. This mechanism employs a crank $(\overline{O_1 O_C})$ to devote the epicycloidal plate gear that orbits about the center (O_1) of the input shaft due to the eccentricity of the shaft. At the same time, the cycloidal plate gear rotates about its own center $(O_{\rm C})$ in the opposite direction of the input shaft, due to the engagement with the stationary ring gear. The resulting motion of the cycloidal plate gear is a compound motion. We can consider that it consists of three links in kinematics: the frame corresponding to $\overline{O_1 O_R}$ (here rollers being attached to the stationary ring gear) as Link 1, the eccentric distance $\overline{O_1 O_C}$ as Link 2, and the cycloid plate gear as Link 3. By Kennedy's theorem, we can easily determine the three instant velocity centers, i.e. a point O_1 as IC₁₂, a point O_C as IC₂₃ and a point M as IC₁₃, respectively, as shown in Fig. 3. Here we will denote the eccentricity $\overline{O_1 O_C}$ corresponding to the eccentric bearing of the input shaft as E, $\overline{O_1M}$ as Q which is an unknown to be determined below, and $\overline{O_1 O_R}$ as R, respectively (Fig. 4). The center distance (or crank length) E, the number of rollers N, the roller radius R_r , and the radius of the roller gear R are usually assigned design parameters. From the definition of the instant center, both links sharing the instant center will have identical velocity at that point. In Fig. 4, the angular velocity ω_2 of the input shaft (Link 2) and the angular velocity ω_3 of the output cycloid plate gear (Link 3) are illustrated in the same direction (counterclockwise). The magnitude of the velocity \vec{V}_{23} of the point IC₂₃ as shown in Fig. 4 can be determined by

$$V_{23} = E\omega_2 = (E - Q)\omega_3.$$
(1)

It means that the actual orientation of ω_2 and ω_3 is in the opposite direction to each other.

Pollitt [11] showed how to find the point of contact between the cycloid plate gear (planetary gear) and the cylindrical rollers which make up the teeth of the stationary ring gear (sun gear)

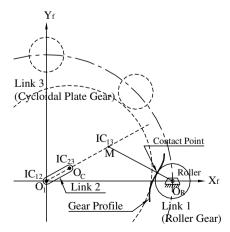


Fig. 3. Instant velocity centers of a stationary ring gear type epicycloid reducer.

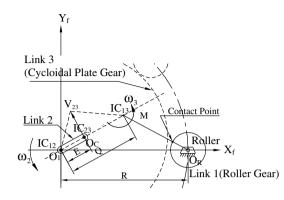


Fig. 4. Velocity at I_{23} of a stationary ring gear type epicycloid reducer.

and concluded that the number of rollers (N) required in the sun gear is one more than the gear ratio (known as the number of lobes, i.e. N - 1). Therefore, the angular velocity ratio m_V defined as the output angular velocity divided by the input angular velocity can be written as

$$m_V = \frac{\omega_3}{\omega_2} = \frac{1}{1 - N}.$$
(2)

From Eqs. (1) and (2), we can easily determine the unknown distance Q as follows

$$Q = EN.$$
(3)

From the rule of the instant velocity center determination in general contact mechanisms, we have already known that the common normal line segment of $\overline{IC_{13}O_R}$ passes through the common tangent line of the contact point between the epicycloid plate gear (Link 3) and the roller gear (Link 1). Therefore, the contact point $C^f(C_x^f, C_y^f)$ in the stationary coordinate system $S_f(x_f, y_f)$ and the corresponding contact angle ψ can be determined from Fig. 5 as below

$$C_{\rm r}^f = R - R_{\rm r} \cos \psi, \quad C_{\rm v}^f = R_{\rm r} \sin \psi, \tag{4}$$

$$\psi = \tan^{-1} \left[\frac{EN \sin \phi_2}{R - EN \cos \phi_2} \right] = \tan^{-1} \left[\frac{\sin \phi_2}{(R/EN) - \cos \phi_2} \right],\tag{5}$$

where R_r is the radius of the roller and ϕ_2 is the rotational input angle of Link 2.

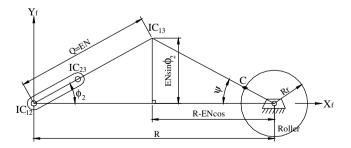


Fig. 5. Contact point of an epicycloidal plate gear and a roller.

We can observe from Eq. (5) that it should be R/EN > 1 (or E < R/N), otherwise the contact angles have discontinuous singularities at some rotation angles (see Fig. 6). Therefore we can obtain the valuable information on the size of eccentric cam of the input shaft with the limitation of E < R/N. It provides that the internal cycloidal plate gear rolls on the stationary ring gear without interference.

Before deriving the profile equation of the epicycloidal plate gear, four coordinate systems corresponding to this speed reducer should be defined as shown in Fig. 7: a stationary reference system $S_f(x_f, y_f)$, and three mobile reference systems $S_2(x_2, y_2)$, $S_3(x_3, y_3)$ and $S_{23}(x_{23}, y_{23})$. The position and the orientation of the reference system S_2 is defined by the input shaft rotation angle ϕ_2 of Link 2, and those of the reference systems S_3 and S_{23} are defined by the cycloidal plate rotation angle ϕ_3 of Link 3.

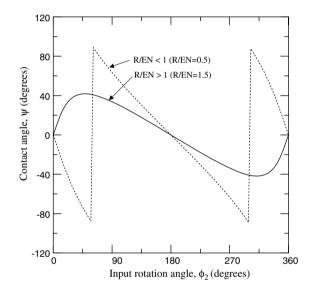


Fig. 6. Contact angle variation in accordance with R/EN.

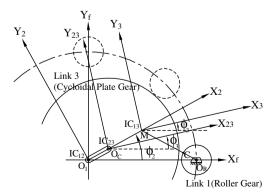


Fig. 7. Coordinate systems for a stationary ring gear type epicycloid reducer.

Generally, the origins of coordinate systems do not coincide and the orientation of the systems is different. In such a case the coordinate transformation may be based on the application of homogeneous coordinates and 4×4 matrices that describe separately rotation about a stationary axis and displacement of one coordinate system with respect to the other [12]. For the homogeneous coordinate transformation from the contact point of C^{f} in S_{f} -reference system to that of C^{23} in S_{23} -reference system with the origin of $O_{\rm C}$, the following matrix equation is defined:

$$C^{23} = M_{23,f}C^f = M_{23,3}M_{3,f}C^f = M_{23,3}M_{3,2}M_{2,f}C^f = M_{23,2}M_{2,f}C^f,$$
(6)

where the matrix $M_{i,j}$ describes transformation S_j to S_i , given by

$$M_{23,2} = \begin{bmatrix} \cos(\phi_2 - \phi_3) & -\sin(\phi_2 - \phi_3) & 0 & -E\cos(\phi_2 - \phi_3) \\ \sin(\phi_2 - \phi_3) & \cos(\phi_2 - \phi_3) & 0 & -E\sin(\phi_2 - \phi_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$M_{2,f} = \begin{bmatrix} \cos\phi_2 & \sin\phi_2 & 0 & 0 \\ -\sin\phi_2 & \cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$(8)$$

$$C^{f} = \begin{bmatrix} R - R_{\rm r} \cos \psi & R_{\rm r} \sin \psi & 0 & 1 \end{bmatrix}^{\rm T},$$

$$\tag{9}$$

where superscript T means the transpose of the matrix.

The resulting expression of Eq. (6) is

$$C^{23} = \begin{bmatrix} R\cos\phi_3 - R_r\cos(\phi_3 + \psi) - E\cos(\phi_2 - \phi_3) \\ -R\sin\phi_3 + R_r\sin(\phi_3 + \psi) - E\sin(\phi_2 - \phi_3) \\ 0 \\ 1 \end{bmatrix}.$$
 (10)

Rewriting Eq. (1), we have $E \frac{d\phi_2}{dt} = (E - Q) \frac{d\phi_3}{dt}$. Then we obtain the following relation with the aid of Eq. (3),

$$\frac{\mathrm{d}\phi_2}{\mathrm{d}\phi_3} = \frac{E-Q}{E} = \frac{1}{m_V} = 1 - N \quad \text{or} \quad \phi_2 = (1-N)\phi_3. \tag{11}$$

If we define $\phi = \phi_3$ by the generated parameter of output motion, we can lead to the following lobe profile equations for this speed reducer from Eq. (10) with the relation of Eq. (11),

$$C_x^{23} = R\cos\phi - R_r\cos(\phi + \psi) - E\cos(N\phi), \qquad (12a)$$

$$C_y^{23} = -R\sin\phi + R_r\sin(\phi + \psi) + E\sin(N\phi), \qquad (12b)$$

where

$$\psi = \tan^{-1} \left[\frac{\sin(1-N)\phi}{(R/EN) - \cos(1-N)\phi} \right] \quad (0^{\circ} \le \phi \le 360^{\circ}).$$
(13)

3. Rotating ring gear type epicycloid reducer

The situation envisaged is the same that of Section 2 but ring gear rotating at constant speed (Fig. 8). It is also modeled kinematically into three-link and three-joint mechanism: the frame corresponding to $\overline{O_1O_C}$ as Link 1, the roller gear attached to the rotating ring gear as Link 2, and the epicycloid plate gear as Link 3. Three instant velocity centers are given by the point O_1 as IC₁₂, the point O_C as IC₁₃ and the point M as IC₂₃, respectively (Fig. 9). From Fig. 9, the speed V_{23} at the point of IC₂₃ and the angular velocity ratio m_V for this epicycloid reducer are given by

$$V_{23} = Q\omega_2 = (Q - E)\omega_3,$$
(14)

$$m_V = \frac{\omega_2}{\omega_3} = \frac{N-1}{N},\tag{15}$$

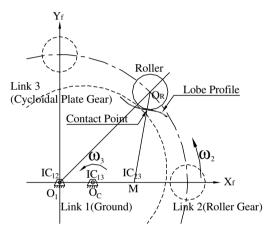


Fig. 8. Instant velocity centers for a rotating ring gear type epicycloid reducer.

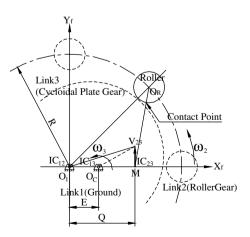


Fig. 9. Velocity of a rotating ring gear type epicycloid reducer at I_{23} .

where the angular velocity ω_3 is the input angular velocity of the epicycloid plate gear and ω_2 is the output angular velocity of the rotating ring gear with rollers. It is noted that ω_2 and ω_3 have the same orientation.

We can determine the unknown distance Q from Eqs. (14) and (15) as follows:

$$Q = EN. (16)$$

After determination of the position of IC₂₃, the contact position C^2 in S₂-reference system and the contact angle ψ can be obtained from Fig. 10 as follows:

$$C_x^2 = R - R_r \cos\psi, \quad C_y^2 = -R_r \sin\psi, \tag{17}$$

$$\psi = \tan^{-1} \left[\frac{\sin \phi_2}{(R/EN) - \cos \phi_2} \right] \tag{18}$$

with the condition of E < R/N.

To obtain the lobe profile in S_3 -reference system (Fig. 11) of the epicycloid plate gear, it is taken the homogeneous coordinate transformation in the form,

$$C^3 = M_{3,2}C^2 = M_{3,f}M_{f,2}C^2, (19)$$

where

$$M_{3,f} = \begin{bmatrix} \cos \phi_3 & \sin \phi_3 & 0 & -E \cos \phi_3 \\ -\sin \phi_3 & \cos \phi_3 & 0 & E \sin \phi_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$M_{f,2} = \begin{bmatrix} \cos \phi_2 & -\sin \phi_2 & 0 & 0 \\ \sin \phi_2 & \cos \phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
(20a)
(20b)

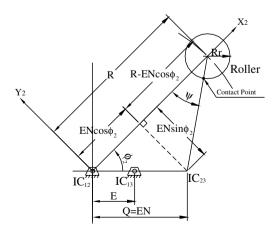


Fig. 10. Contact point between an epicycloidal plate gear and a roller.

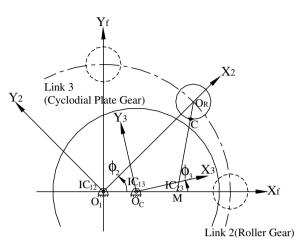


Fig. 11. Coordinate systems for a rotating ring gear type epicycloid reducer.

$$C^{2} = \begin{bmatrix} R - R_{\rm r} \cos\psi & -R_{\rm r} \sin\psi & 0 & 1 \end{bmatrix}^{\rm T}.$$
(21)

The result of the matrix Eq. (19) is given by

$$C^{3} = \begin{bmatrix} R\cos(\phi_{3} - \phi_{2}) - R_{r}\cos(\phi_{3} - \phi_{2} - \psi) - E\cos\phi_{3} \\ -R\sin(\phi_{3} - \phi_{2}) + R_{r}\sin(\phi_{3} - \phi_{2} - \psi) + E\sin\phi_{3} \\ 0 \\ 1 \end{bmatrix}.$$
 (22)

From Eqs. (14) and (16), we can obtain the following relation, given by

$$\phi_2 = \frac{N-1}{N}\phi_3. \tag{23}$$

If we define ϕ by the generated parameter of motion, we obtain the relation of $\phi_3 = N\phi$, and hence $\phi_2 = (N - 1)\phi$. Therefore, we can rewrite Eq. (22) for the lobe profile equations of the current epicycloid plate gear in the forms,

$$C_x^3 = R\cos\phi - R_r\cos(\phi - \psi) - E\cos(N\phi), \qquad (24a)$$

$$C_{\nu}^{3} = -R\sin\phi + R_{\rm r}\sin(\phi - \psi) + E\sin(N\phi), \qquad (24b)$$

where

$$\psi = -\tan^{-1} \left[\frac{\sin(1-N)\phi}{(R/EN) - \cos(1-N)\phi} \right] \quad (0^{\circ} \leqslant \phi \leqslant 360^{\circ}).$$

$$(25)$$

It should be noted from Eqs. (12) and (24) that the obtained lobe profile equations of the two epicycloid reducers are exactly the same forms.

4. Stationary ring gear type hypocycloid reducer

In Fig. 12, the stationary ring gear type hypocycloid reducer is depicted schematically. We can regard the frame corresponding to a stationary hypocycloid plate gear (ring gear) as Link 1, the eccentric distance $\overline{O_C O_{RG}}$ as Link 2, and a roller gear as Link 3, respectively. The three instant velocity centers are given at the point O_C as IC₁₂, the point O_{RG} as IC₂₃, and the point M as IC₁₃, respectively, as shown in Fig. 13. Here we denote the distances $\overline{O_C O_{RG}}$ as E, $\overline{O_C M}$ as Q and $\overline{O_{RG} O_R}$ as R, respectively. The speed V_{23} at IC₂₃ can be written from Fig. 13,

$$V_{23} = E\omega_2 = (E - Q)\omega_3, \tag{26}$$

where the angular velocities ω_2 and ω_3 , the direction of which are opposite to each other, represent the input angular velocity by the input shaft and the output angular velocity of the roller gear, respectively.

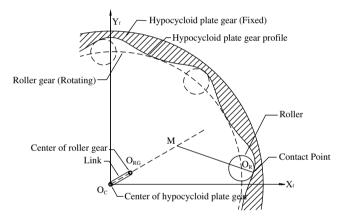


Fig. 12. Instant velocity centers of a stationary ring gear type hypocycloid reducer.

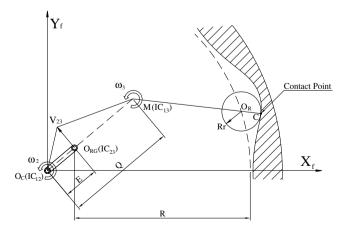


Fig. 13. Velocity at instant velocity center I_{23} .

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In this case, the angular velocity ratio m_V is given by

$$m_V = \frac{\omega_3}{\omega_2} = -\frac{1}{N}.$$
(27)

So the position of Q is determined by Eqs. (26) and (27) as

$$Q = E(N+1). \tag{28}$$

To determine the contact position and contact angle, a detailed schematic is shown in Fig. 14. By the given figure, the contact point in S_{23} -coordinate system and the contact angle ψ are determined as follows

$$C_x^{23} = R + R_r \cos \psi, \quad C_y^{23} = -R_r \sin \psi,$$
 (29)

$$\psi = \tan^{-1} \left[\frac{\sin(\phi_2 - \phi_3)}{(R/EN) - \cos(\phi_2 - \phi_3)} \right] \quad (E < R/N),$$
(30)

where the angles ϕ_2 and ϕ_3 are the input rotation angle of the input shaft and the output rotation angle of the roller gear, respectively. In order to transform C^{23} to C^{f} (Fig. 15), we take a following matrix equation

$$C^{f} = M_{f,23}C^{23} = M_{f,3}M_{3,23}C^{23} = M_{f,2}M_{2,3}M_{3,23}C^{23} = M_{f,2}M_{2,23}C^{23},$$
(31)

where the transformation matrices $M_{f,2}$ and $M_{2,23}$, and C^{23} matrix are given by

$$M_{f,2} = \begin{bmatrix} \cos \phi_2 & -\sin \phi_2 & 0 & 0\\ \sin \phi_2 & \cos \phi_2 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix},$$
(32)

$$M_{2,23} = \begin{bmatrix} \cos(\phi_3 - \phi_2) & -\sin(\phi_3 - \phi_2) & 0 & E\\ \sin(\phi_3 - \phi_2) & \cos(\phi_3 - \phi_2) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix},$$
(33)

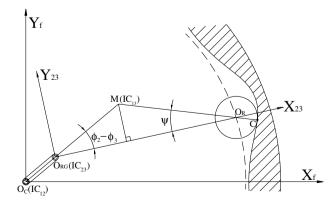


Fig. 14. Contact point of a hypocycloid plate gear and a roller.

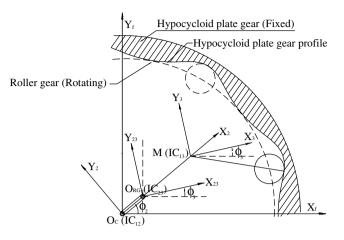


Fig. 15. Coordinate systems of a stationary ring gear type hypocycloid reducer.

$$C^{23} = [R + R_{\rm r} \cos \psi - R_{\rm r} \sin \psi \quad 0 \quad 1]^{\rm T},$$
(34)

so it leads to

$$C^{f} = \begin{bmatrix} R\cos\phi_{3} + R_{r}\cos(\phi_{3} - \psi) + E\cos\phi_{2} \\ R\sin\phi_{3} + R_{r}\sin(\phi_{3} - \psi) + E\sin\phi_{2} \\ 0 \\ 1 \end{bmatrix}.$$
(35)

If we define $\phi = \phi_3$ by the generated parameter of motion, we have the following relationship from Eqs. (26)–(28),

$$\phi_2 = -N\phi. \tag{36}$$

Finally, we obtain the lobe profile equations for a stationary type hypocycloid plate gear in the forms,

$$C'_{x} = R\cos\phi + R_{\rm r}\cos(\phi - \psi) + E\cos(N\phi), \tag{37a}$$

$$C_{\nu}^{f} = R\sin\phi + R_{\rm r}\sin(\phi - \psi) - E\sin(N\phi), \qquad (37b)$$

where

$$\psi = -\tan^{-1} \left[\frac{\sin(N+1)\phi}{(R/EN) - \cos(N+1)\phi} \right] \quad (0^{\circ} \leqslant \phi \leqslant 360^{\circ}).$$
(38)

5. Rotating ring gear type hypocycloid reducer

Fig. 16 shows a schematic of the rotating ring gear type hypocycloid reducer. It also consists of three links and three joints kinematically: the frame corresponding to $\overline{O_C O_{RG}}$ as Link 1, the rotating hypocycloid ring gear as Link 2, the internal roller gear as Link 3. Three instant velocity

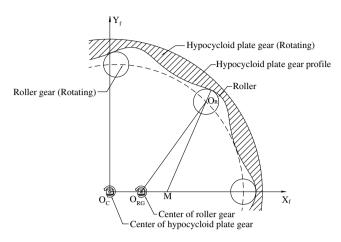


Fig. 16. Instant velocity centers of a rotating ring gear type hypocycloid reducer.

centers are given at a point O_C as IC₁₂, a point O_{RG} as IC₁₃ and a point M as IC₂₃, respectively. From Fig. 17, the speed V_{23} at IC₂₃ and the angular velocity ratio m_V for a rotating ring gear type hypocycloid reducer can be written as

$$V_{23} = Q\omega_2 = (Q - E)\omega_3,$$
 (39)

$$m_V = \frac{\omega_2}{\omega_3} = \frac{N}{N+1},\tag{40}$$

where ω_3 is the input angular velocity of the roller gear, and ω_2 is the output angular velocity of the hypocycloid plate gear. It is noted that ω_3 and ω_2 have the same rotational direction.

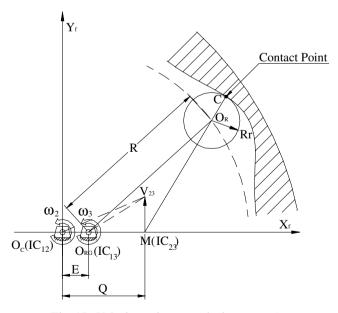


Fig. 17. Velocity at instant velocity center I_{23} .

We can determine the unknown distance Q of the position IC₂₃ from Eqs. (39) and (40) as follows

$$Q = E(N+1). \tag{41}$$

Similarly, the contact position C^3 in S_3 -reference system and the contact angle ψ can be obtained from Fig. 18 as follows

$$C_x^3 = R + R_r \cos\psi, \quad C_y^3 = R_r \sin\psi, \tag{42}$$

$$\psi = \tan^{-1} \left[\frac{\sin \phi_3}{(R/EN) - \cos \phi_3} \right] \tag{43}$$

with E < R/N.

Our next goal is to represent the contact point C^3 in S_2 -reference system (Fig. 19). The coordinate transformation S_3 to S_2 is based on the following matrix equation

$$C^2 = M_{2,f} M_{f,3} C^3, (44)$$

where

$$M_{2,f} = \begin{bmatrix} \cos \phi_2 & \sin \phi_2 & 0 & 0 \\ -\sin \phi_2 & \cos \phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$M_{f,3} = \begin{bmatrix} \cos \phi_3 & -\sin \phi_3 & 0 & E \\ \sin \phi_3 & \cos \phi_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
(45)

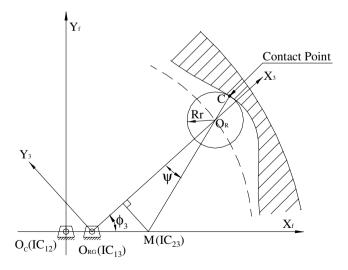


Fig. 18. Contact point of a rotating hypocycloid plate ring gear and a roller.

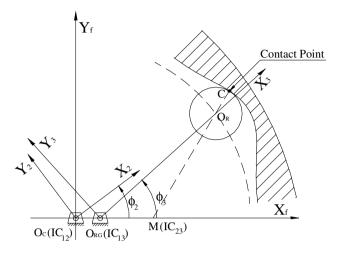


Fig. 19. Coordinate systems of a rotating ring gear type hypocycloid reducer.

$$C^{3} = \begin{bmatrix} R + R_{\rm r} \cos\psi & R_{\rm r} \sin\psi & 0 & 1 \end{bmatrix}^{\rm T}, \tag{47}$$

so the result of the matrix Eq. (44) can be written as

$$C^{2} = \begin{bmatrix} (R + R_{\rm r}\cos\psi)\cos(\phi_{2} - \phi_{3}) + R_{\rm r}\sin\psi\sin(\phi_{2} - \phi_{3}) + E\cos\phi_{2} \\ -(R + R_{\rm r}\cos\psi)\sin(\phi_{2} - \phi_{3}) + R_{\rm r}\sin\psi\cos(\phi_{2} - \phi_{3}) - E\cos\phi_{2} \\ 0 \\ 1 \end{bmatrix}.$$
 (48)

If we define ϕ by the generated parameter of motion, we obtain the relation of $\phi_2 = N\phi$ with the aid of Eqs. (39)–(41), and $\phi_3 = (N+1)\phi$. Therefore, we can rewrite Eq. (48) for the lobe profile equations of the rotating hypocycloid plate ring gear in the forms,

$$C_x^2 = R\cos\phi + R_r\cos(\phi + \psi) + E\cos(N\phi), \tag{49a}$$

$$C_y^2 = R\sin\phi + R_r\sin(\phi + \psi) - E\sin(N\phi), \tag{49b}$$

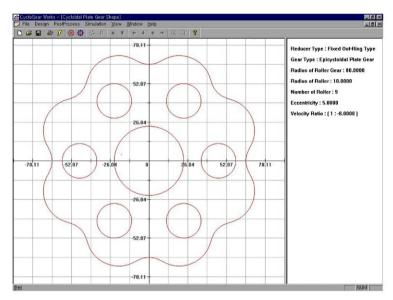
where

$$\psi = \tan^{-1} \left[\frac{\sin(N+1)\phi}{(R/EN) - \cos(N+1)\phi} \right] \quad (0^{\circ} \leqslant \phi \leqslant 360^{\circ}).$$
(50)

6. Design examples

A computer program based on C++ language is developed to design the cycloid reducers. This CAD program has the characteristics of the graphic user interface and the simulation of the real operation for the 4 types of cycloid reducers.

Fig. 20 shows the design example of a stationary ring gear type epicycloidal reducer. The speed ratio equals to -1/8, R = 80 mm, $R_r = 10$ mm, N = 9, and eccentricity E = 5 mm which is to be determined under the condition of E < R/N. It shows that the epicycloidal plate gear rolls on all rollers without interference. In Fig. 21, a design example of a rotating ring gear type epicycloidal reducer is demonstrated. In the case, the speed ratio equals to 14/15, R = 120 mm, $R_r = 9$ mm,





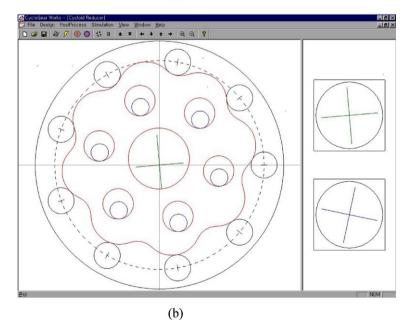
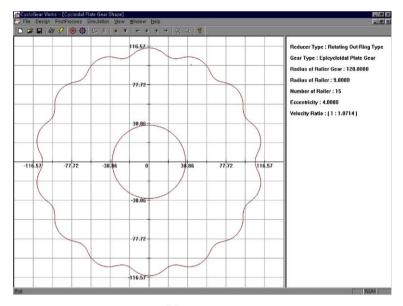
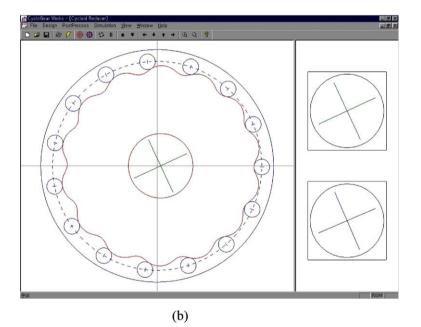
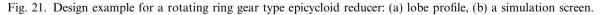


Fig. 20. Design example for a stationary ring gear type epicycloid reducer: (a) lobe profile, (b) a simulation screen.



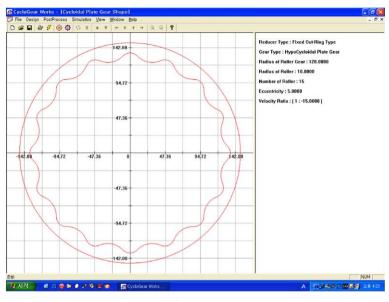






N = 15, and E = 4 mm. It also shows that the epicycloidal plate gear rolls on all rollers without interference.

Fig. 22 corresponds to the design example of a stationary ring gear type hypocycloidal reducer. The speed ratio equals to -1/15, R = 120 mm, $R_r = 10$ mm, N = 15, and E = 5 mm. From the figure, we can observe that all roller gears roll on the hypocycloidal plate ring gear without





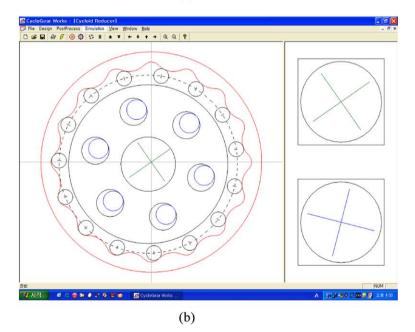
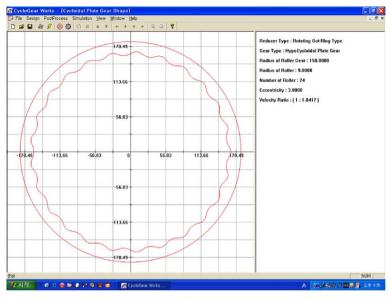


Fig. 22. Design example for a stationary ring gear type hypocycloid reducer: (a) lobe profile, (b) a simulation screen.

interference. Finally, a design example of a rotating ring gear type hypocycloidal reducer is demonstrated in Fig. 23. The speed ratio equals to 24/25, R = 150 mm, $R_r = 9$ mm, N = 24, and E = 3 mm. It also shows that the roller gear rolls on the hypocycloidal ring gear smoothly. Therefore, we can confirm via the above examples that the presented design approach for cycloid reducers is exact and easy to understand.





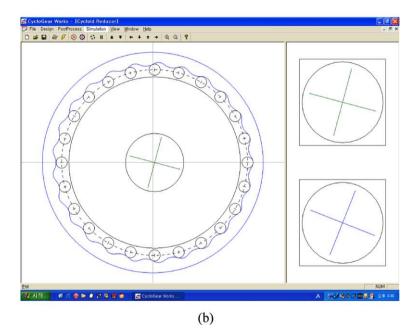


Fig. 23. Design example for a rotating ring gear type hypocycloid reducer: (a) lobe profile, (b) a simulation screen.

7. Conclusions

Because of meshing with the rollers in all lobes at any one time, the cycloid reducer has the peculiar lobe profile. In this study, the lobe profiles for the cycloid reducers have been analyzed by the principle of the instant velocity center and the homogeneous coordinate transformation technique. The following conclusions can be drawn:

- 1. The lobe profiles for the four types of the cycloid reducers are considered and analyzed. The present results are easy to understand and exact.
- 2. The center distance E should be less than R/N in all cases, which provides the condition of no interference.
- 3. The obtained lobe profile equation is exactly the same form irrespective of whether the ring gear rotates or not.
- 4. The developed design methodology has been successfully applied to cycloid speed reducers using a computer-aided program, and some examples have been presented to verify the validity of the developed methodology.

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