CHAPTER 3
MODIFIED CAM CURVES

Harold A. Rothbart, D.Eng.

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SYMBOLS

h = total rise of the follower, in
h' = maximum rise of follower segment, in
h = maximum follower displacement for full or half curve, in
t = time, sec.
y = follower displacement
$y' = \dfrac{dy}{d\theta}$ = follower velocity, dimensionless
$y'' = \dfrac{d^2y}{d\theta^2}$ = follower acceleration, dimensionless
$y''' = \dfrac{d^3y}{d\theta^3}$ = follower jerk, dimensionless
$\ddot{y} = \dfrac{d^2y}{dt^2} = \omega^2 y''$ = follower acceleration
$\dot{y} = \dfrac{dy}{dt} = \omega y'$ = follower velocity
$\dddot{y} = \dfrac{d^3y}{dt^3} = \omega^3 y'''$ = follower jerk
A = follower acceleration, in/sec^2
V_{max} = maximum velocity, ips
V_0 = follower initial velocity, in/sec
a = follower acceleration, dimensionless
β = cam angle for rise h, radians
β = angle for maximum follower displacement, radians
β_1, β_2 = periods during positive and negative accelerations respectively, radians
θ = cam angle rotation, radians
ω = cam speed, rad/sec
v_0 = follower initial velocity, in/radians

3.1 INTRODUCTION

In Chap. 2, the characteristics of displacement, velocity, acceleration, and jerk of basic symmetrical curves were presented. These curves were employed because of their simplicity of mathematical analysis and ease of construction. However, for many machine performance requirements, as when the cam requires either special functional motions or must operate at high speeds, the basic symmetrical curves are inadequate and modifications in curve selection are necessary. These modifications can consist of blending, skewing, or combining sectors of the cubic curves, simple harmonic curves, cycloidal curves, constant velocity curves, and constant acceleration curves.

Last, it should be mentioned that cam curve development (not shown) can be accomplished by starting with the fourth derivative ($d^{if}y/dy^{if}$) curve with numerical trial and error combined with past experience to find the ultimate desired cam shape. Computers are employed to perform the increment integration in determining the displacement velocity, acceleration, and jerk curves.

Curve development and selection is one of the primary steps in the design of any cam-follower system. Later chapters include investigation of the pressure angle, cam curvature, cam torques, lubrication, materials, and necessary fabrication tolerances, among other things. A typical design evolutionary process will proceed as a series of trade-offs to produce the final design.

This chapter has two parts:

- complete mathematical development for popular DRD curves
- a simplified procedure for combining sectors of various basic curves

The dwell-rise-dwell (DRD) and dwell-rise-return-dwell (DRRD) curves will be analyzed. The rise-return-rise (RRR) curve is not presented since the *eccentric mechanism* of Chap. 15 satisfies the action in a simple, reliable, less expensive way. The RRR curve is also best for high-speed requirements because it provides a motion curve having continuity in all of its derivatives.

Note that the DRD cam curve is a portion of the total action which could be a part of the dwell-rise-dwell-return-dwell (DRDRD) cam. Figure 3.1 shows the complete cycle of blended DRD cycloidal curve producing a DRDRD cam. The period of rise is smaller than the period of fall, producing higher rise maximum acceleration than the maximum fall acceleration.

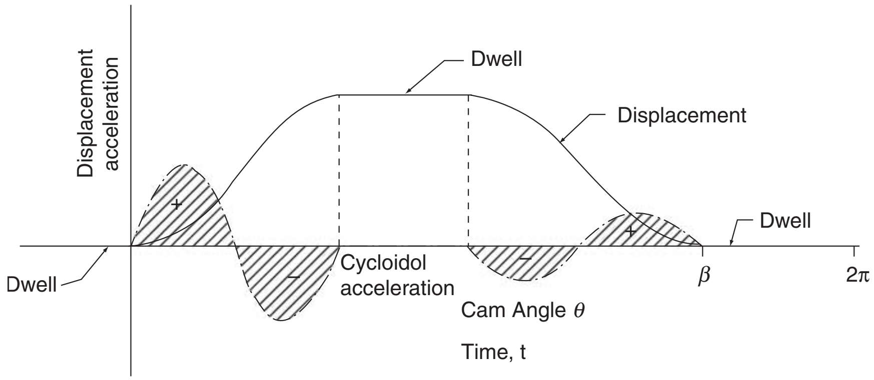


FIGURE 3.1. Dwell-rise-dwell-return-dwell cam with cycloidal acceleration curves.

MODIFIED CAM CURVES

3.2 FUNDAMENTALS

In this section, the fundamental conditions for shaping and combining curves are presented, and typical combinations of simple curves are shown. It is not practical to show all the possibilities, but it should be noted that any combination of sections of basic curves may be utilized to fulfill design requirements. The control conditions are:

- The sum of the displacements of the combining sectors shall equal the total rise of the follower.
- The sum of angles rotated of the combining sectors shall equal the total cam angle.
- The velocities at each sector junctions shall be equal.
- High-speed action requires that acceleration at all sector junctions be equal, that is, having no discontinuity in the acceleration and no infinite jerk value. Also, the acceleration curve should have the lowest maximum value with the value of jerk not too large.
- Sometimes special design requirements dictate the proportions of the acceleration curve. An example may be the controlling of the ratio of positive and negative acceleration periods and shapes. An asymmetrical acceleration curve, with the maximum positive acceleration larger than the negative maximum acceleration (ratio about 3 : 1) would be a good choice for spring-loaded high-speed cams. Smaller springs, larger cam curvatures, and longer surface life result.

On rare occasions limitations in the available manufacturing facilities may dictate the cam profile developed. For convenience, we have presented Fig. 3.2, which gives a comparison of important cam curves. Note that the velocity, acceleration, and jerk curves presented in this figure are all normalized (i.e., they have a unit total displacement h in a unit cam displacement β).

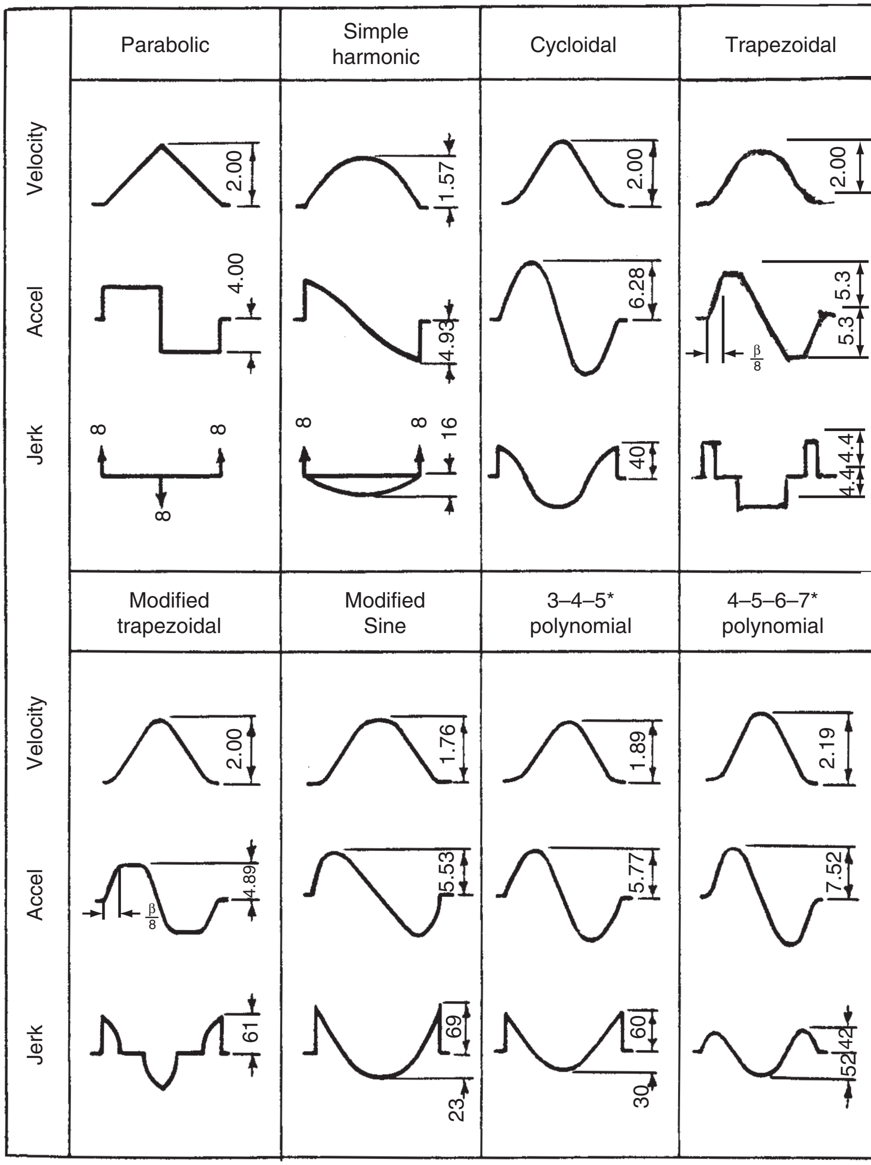
To illustrate terminology we see that the trapezoidal acceleration curve (discussed later) is a continuous function whereas its derivative (jerk curve) has many discontinuities. Note that continuity of the jerk curve is of little value due to the usual tolerance limitations of cam profile machine tool fabrication.

3.3 MODIFIED CONSTANT VELOCITY CURVE

In Chap. 2, we saw that the simplest curve is the constant velocity curve. It has a straight-line displacement at a constant slope. It also has the smallest cam for a given rise and provides a long stroke action. In this section we will blend any acceptable curve at the dwell ends for proper rise. The cycloidal curve or parabolic curves have been utilized depending on the cam speed, mass of the follower, and work performed by the machine.

As an example let us combine the parabolic motion curve blended with the straight-line displacement curve.

EXAMPLE *A cam having a rigid heavy-mass follower rotates at 300 rpm with a total DRD rise of 4 inches in 130 degrees of cam movement. As a preliminary study, use a parabolic curve modified with the constant velocity curve. The action is as follows: (a) for the first 40 degrees a positive parabolic curve acceleration; (b) for the next 30 degrees a straight-line displacement; and (c) for the last 60 degrees a negative parabolic curve acceleration. Find the ratio of accelerations, and plot all characteristic curves indicating pertinent values.*



*In Chap. 4

FIGURE 3.2. Comparison of important DRD cam curves.

Solution *The time for a revolution = 60/300 = 0.2 sec/rev. Let us divide the total action into three parts, as shown in Fig. 3.3 with T and Q the tangent points of the curves, giving the times*

$$t_1 = 0.2\left(\frac{40}{360}\right) = 0.0222 \text{ sec}$$

$$t_2 = 0.2\left(\frac{30}{360}\right) = 0.0167 \text{ sec}$$

$$t_3 = 0.2\left(\frac{60}{360}\right) = 0.0333 \text{ sec}$$

From the last chapter, we have the constant acceleration displacement

$$y = V_0 t + \frac{1}{2} A t^2 \text{ in} \tag{2.24}$$

where V_0 = initial velocity, in./sec
t = time, sec
A = acceleration, in./sec^2

Therefore, the displacements and the velocity for the parabolic motion of parts 1 and 3 are

$$\begin{aligned} y_1 &= \frac{1}{2} A_1 t_1^2 \\ y_3 &= v_Q t_3 + \frac{1}{2} A_3 t_3^2 \\ v_T &= A_1 t_1 \\ v_Q &= -A_3 t_3 \end{aligned} \tag{3.1}$$

For constant velocity of part 2

$$v_Q = v_T.$$

Combining yields

$$A_1 t_1 = -A_3 t_3.$$

Substituting yields

$$A_1(0.0222) = -A_3(0.0333)$$

or

$$A_1 = -1\frac{1}{2} A_3.$$

Also, the displacement of part 2 is

$$y_2 = v_T t_2.$$

We are given that the total displacement

$$y_1 + y_2 + y_3 = 4.$$

Substituting gives

$$\frac{1}{2}A_1t_1^2 + v_Tt_2 + V_Qt_3 + \frac{1}{2}A_3t_3^2 = 4.$$

Again substituting,

$$\frac{1}{2}A_1t_1^2 + A_1t_1t_2 + A_1t_1t_3 - \frac{1}{3}A_1t_3^2 = 4$$

$$\frac{1}{2}A_1(0.0222)^2 + A_1(0.0222)(0.0167) + A_1(0.0222)(0.0333) - \frac{1}{3}A_1(0.0333)^2 = 4.$$

Thus

$$A_1 = 4050 \text{ in/sec}^2$$
$$A_3 = -2700 \text{ in/sec}^2$$

The velocity at the points of intersection from Fig. 3.3 is

$$v_T = v_Q = A_1t_1 = (4050)(0.0222) = 90 \text{ ips}.$$

The displacements for each part are

$$y_1 \frac{1}{2}A_1t_1^2 = \frac{1}{2}(4050)(0.0222)^2 = 1 \text{ in}$$
$$y_2A_1t_1t_2 = (4050)(0.0222)(0.0167) = 1.5 \text{ in}$$
$$y_3 = 1.5 \text{ in}$$

We can now plot the curves of displacement, velocity, and acceleration as shown in Fig. 3.3.

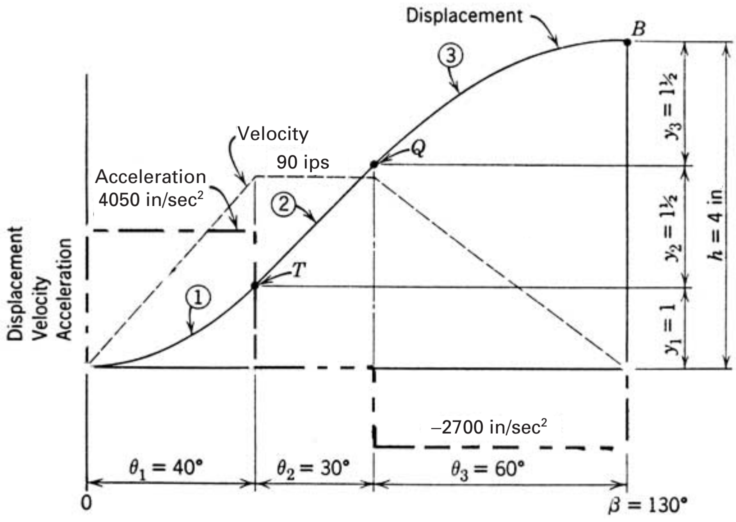


FIGURE 3.3. Example modified constant velocity curve.

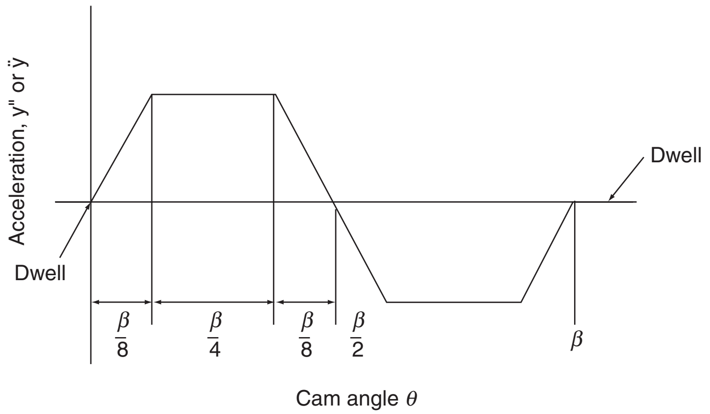


FIGURE 3.4. Trapezoidal acceleration curve—DRD cam.

3.4 TRAPEZOIDAL CURVE

The trapezoidal acceleration curve is a combination of the cubic and parabolic curves. It modifies the parabolic curve by changing its acceleration from a rectangular to a trapezoidal shape. It is an early composite that was first recognized by Neklutin (1969). He showed that the trapezoidal acceleration curve is an improvement over the parabolic curve and that it offers good dynamic response under high-speed operation. It is a slight improvement over the cycloidal curve with its lower maximum acceleration.

In trapezoidal curve motion, the fraction of the total rise angle used for the initial cubic segment is known as the *b* value for the motion. In Fig. 3.4 we see a trapezoidal acceleration curve (DRD cam) where $b = 1/8$. This choice of $b = 1/8$ yields satisfactory cam-follower performance.

3.5 MODIFIED TRAPEZOIDAL CURVE

A combination cam curve (Chen, 1982) that has been used in lieu of the trapezoidal acceleration curve is the modified trapezoidal curve. It is composed of a parabolic motion combined with the cycloidal curve. This combination reduces the maximum acceleration at the expense of somewhat higher jerk values.

The modified trapezoidal curve is popular in industry. However, it has one objectionable characteristic: the torque (discussed in later chapters) goes from positive maximum to negative maximum in 20 percent of the travel time. If dynamic forces represent a significant part of the load on the cam, this sudden release of energy may be detrimental to the cam-follower system performance and limit the operating speeds. Much better torque characteristics can be obtained with the modified sine curve (see Sec. 3.7).

Figure 3.5*a* shows the basic cycloidal curve from which the modified trapezoidal curve is developed. The displacement and acceleration diagrams of the modified trapezoidal are also shown. The variables pertaining to the cycloidal curve are denoted by the primed symbols. At the start of the rise from *A* to *B* (Fig. 3.5*b*) the follower acceleration is a quarter sine wave; from *B* to *C* the acceleration is constant; and from *C* to *D* the acceleration decreases to zero with a quarter sine wave. After *D*, the follower has negative acceleration in the same way that it was positively accelerated.

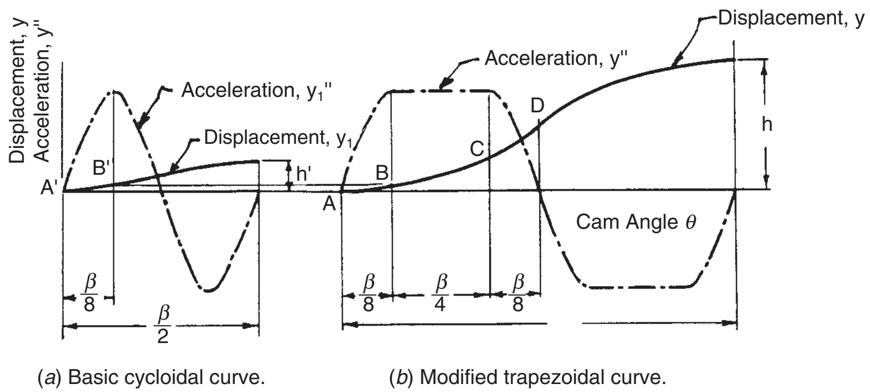


FIGURE 3.5. Modified trapezoidal acceleration curve.

Similar to the trapezoidal acceleration curve, we use $b = {}^1\!/_8$ in the derivation. The equations of the curve from A to B are

$$\begin{aligned} &\text{Displacement} \quad y = h'\left(\frac{2\theta}{\beta} - \frac{1}{2\pi}\sin 4\pi\frac{\theta}{\beta}\right), \\ &\text{Velocity} \quad y' = \frac{h'}{\beta}\left(2 - 2\cos 4\pi\frac{\theta}{\beta}\right), \\ &\text{Acceleration} \quad y'' = \frac{8\pi h'}{\beta^2}\sin 4\pi\frac{\theta}{\beta}, \end{aligned} \tag{3.2}$$

where h' = maximum rise for cycloidal segment

When point B is reached, $\theta = \dfrac{\beta}{8}$. Substituting this value of θ into Eq. (3.2) provides the characteristic equations at point B.

$$y_1 = h'\left(\frac{1}{4} - \frac{1}{2\pi}\right)$$

$$y_1' = \frac{2h'}{\beta}$$

$$y_1'' = \frac{8\pi h'}{\beta^2}.$$

The general expression for displacement under constant acceleration has a displacement

$$y = v_0\theta + \frac{1}{2}a\theta^2 \tag{2.24}$$

where v_0 = follower initial velocity, dimensionless
a = follower acceleration, dimensionless
θ = cam angle rotation, radians

Therefore, the general equations of the curve from B to C are

$$y = y_1 + v_0\left(\theta - \frac{\beta}{8}\right) + \frac{1}{2}a\left(\theta - \frac{\beta}{8}\right)^2$$
$$y' = v_0 + a\left(\theta - \frac{\beta}{8}\right)$$
$$y'' = a.$$

To get displacement, velocity, and acceleration to match at the junction B, it is necessary that

$$v_0 = \frac{2h'}{\beta}$$
$$a = \frac{8\pi h'}{\beta^2}$$

Therefore, the equations from B to C are

$$y = h'\left(\frac{1}{4} - \frac{1}{2\pi}\right) + \frac{2h'}{\beta}\left(\theta - \frac{\beta}{8}\right) + \frac{4\pi h'}{\beta^2}\left(\theta - \frac{\beta}{8}\right)^2 \tag{3.3}$$
$$y' = \frac{2h'}{\beta} + \frac{8\pi h'}{\beta^2}\left(\theta - \frac{\beta}{8}\right)$$
$$y'' = \frac{8\pi h'}{\beta^2}.$$

When point C is reached, $\theta = \frac{3}{8}\beta$. Substituting in Eq. (3.3), we obtain

$$y = \frac{3}{4}h' - \frac{h'}{2\pi} + \frac{\pi h'}{4}$$
$$\therefore y_2 = y - y_1 = \frac{h'}{2} - \frac{\pi h'}{4}$$

The cycloidal displacement is the sum of a constant velocity displacement and a quarter sine wave displacement. The displacement equation of the curve from C to D is

$$y = y_1 + y_2 + C_1 + C_2\frac{\theta - \frac{3}{8}\beta}{\beta} + C_3 \sin\left(4\pi\frac{\theta - \frac{\beta}{2}}{\beta}\right).$$

Hence,

$$y' = \frac{C_2}{\beta} + C_3\frac{4\pi}{\beta}\cos 4\pi\frac{\theta - \frac{\beta}{2}}{\beta}$$
$$y'' = -C_3\frac{16\pi^2}{\beta^2}\sin 4\pi\frac{\theta - \frac{\beta}{2}}{\beta} \tag{3.4}$$

where C_1, C_2, and C_3 are undetermined coefficients that can be obtained as follows:

An acceleration match at point $C\left(\text{when } \theta = \frac{3}{8}\beta\right)$ requires

$$\frac{8\pi h'}{\beta^2} = \left[-C_3 \frac{16\pi^2}{\beta^2} \sin 4\pi \frac{\theta - \frac{\beta}{2}}{\beta}\right]_{\text{at } \theta=\frac{3}{8}\beta}$$

thus giving

$$C_3 = \frac{h'}{2\pi}.$$

A velocity match at point C requires

$$\left[\frac{2h'}{\beta} + \frac{8\pi h'}{\beta^2}\left(\theta - \frac{\beta}{8}\right)\right]_{\theta=\frac{3\beta}{8}} = \left[\frac{C_2}{\beta} + \frac{h'}{2\pi}\frac{4\pi}{\beta}\cos 4\pi \frac{\theta - \frac{\beta}{2}}{\beta}\right]_{\theta=\frac{3}{8}\beta}$$

from which we obtain

$$C_2 = 2h'(1+\pi).$$

The total displacement at point C is

$$y = y_1 + y_2$$

or equivalently

$$\left[C_1 = -2h'(1+\pi)\frac{\theta - \frac{3}{8}\beta}{\beta} - \frac{h'}{2\pi}\sin 4\pi \frac{\theta - \frac{\beta}{2}}{\beta}\right]_{\theta=\frac{3}{8}\beta} = 0,$$

from which C_1 can be obtained

$$C_1 = \frac{h'}{2\pi}.$$

Substituting the values of C_1, C_2, and C_3 in the displacement equation gives the curve from C to D

$$y = \left(\frac{h'}{4} - \frac{h'}{2\pi}\right) + \left(\frac{h'}{2} + \frac{\pi h'}{4}\right) + \frac{h'}{2\pi} + 2h'(1+\pi)\frac{\theta - \frac{3}{8}\beta}{\beta} - \frac{h'}{2\pi}\sin 4\pi \frac{\theta - \frac{\beta}{4}}{\beta}$$

or

$$y = h'\left[-\frac{\pi}{2} + 2(1+\pi)\frac{\theta}{\beta} - \frac{1}{2\pi}\sin 4\pi \frac{\theta - \frac{\beta}{4}}{\beta}\right]. \tag{3.5}$$

At point $D\left(\text{i.e., when } \theta = \dfrac{\beta}{2}\right)$ the total displacement can be found from this equation to be

$$y = h'\left(1 + \frac{\pi}{2}\right).$$

The displacement of the final segment is

$$y_3 = y - y_1 - y_2$$

or

$$y_3 = h'\left(\frac{1}{4} + \frac{\pi}{4} + \frac{1}{2\pi}\right).$$

From the relationship

$$y_1 + y_2 + y_3 = \frac{h}{2},$$

we establish the relationship between h' and h

$$h' = \frac{h}{2+\pi}. \tag{3.6}$$

Therefore, the displacement equations of the first three segments are

$$\begin{aligned}
y &= \frac{h}{2+\pi}\left(\frac{2\theta}{\beta} - \frac{1}{2\pi}\sin 4\pi\frac{\theta}{\beta}\right) && 0 \le \theta \le \frac{\beta}{8} \\
y &= \frac{h}{2+\pi}\left[\frac{1}{4} - \frac{1}{2\pi} + \frac{2}{\beta}\left(\theta - \frac{\beta}{8}\right) + \frac{4\pi}{\beta^2}\left(\theta - \frac{\beta}{8}\right)^2\right] && \frac{\beta}{8} \le \theta \le \frac{3}{8}\beta \\
y &= \frac{h}{2+\pi}\left[-\frac{\pi}{2} + 2(1+\pi)\frac{\theta}{\beta} + \frac{1}{2\pi}\sin 4\pi\frac{\theta - \frac{\beta}{2}}{\beta}\right] && \frac{3}{8}\beta \le \theta \le \frac{\beta}{2}.
\end{aligned} \tag{3.7}$$

Evaluating all constants, characteristic curve equations are:

for $\quad \theta \le \dfrac{\theta}{\beta} \le \dfrac{1}{8}$, (3.8)

$$\begin{aligned}
y &= 0.09724613h\left(4\frac{\theta}{\beta} - \frac{1}{\pi}\sin 4\pi\frac{\theta}{\beta}\right) \\
y' &= 0.3889845\frac{h}{\beta}\left(1 - \cos 4\pi\frac{\theta}{\beta}\right) \\
y'' &= 4.888124\frac{h}{\beta^2}\sin 4\pi\frac{\theta}{\beta} \\
y''' &= 61.425975\frac{h}{\beta^3}\cos 4\pi\frac{\theta}{\beta}
\end{aligned}$$

for $\frac{1}{8} \le \frac{\theta}{\beta} \le \frac{3}{8}$,

$$y = h\left(2.444016188\frac{\theta^2}{\beta} - 0.22203094\frac{\theta}{\beta} + 0.00723406\right)$$

$$y' = \frac{h}{\beta}\left(4.888124\frac{\theta}{\beta} - 0.222031\right)$$

$$y'' = \frac{h}{\beta^2}(4.88124)$$

$$y''' = 0$$

for $\frac{3}{8} \le \frac{\theta}{\beta} \le \frac{1}{2}$,

$$y = h\left(1.6110155\frac{\theta}{\beta} - 0.0309544\sin\left(4\pi\frac{\theta}{\beta}\right) - 0.3055077\right)$$

$$y' = \frac{h}{\beta}\left(1.6110155 - 0.3889845\cos\left(4\pi\frac{\theta}{\beta}\right)\right)$$

$$y'' = \frac{h}{\beta^2}\left(4.88124\sin\left(4\pi\frac{\theta}{\beta}\right)\right)$$

$$y''' = -61.425975\frac{h}{\beta^3}\cos\left(4\pi\frac{\theta}{\beta}\right)$$

for $\frac{1}{2} \le \frac{\theta}{\beta} \le \frac{5}{8}$,

$$y = h\left(1.6110155\frac{\theta}{\beta} + 0.0309544\sin\left(4\pi\frac{\theta}{\beta}\right) - 0.3055077\right)$$

$$y' = \frac{h}{\beta}\left(1.6110154 + 0.3889845\cos\left(4\pi\frac{\theta}{\beta}\right)\right)$$

$$y'' = \frac{h}{\beta^2}\left(-4.88124\sin\left(4\pi\frac{\theta}{\beta}\right)\right)$$

$$y''' = -61.425975\frac{h}{\beta^3}\left(4\pi\frac{\theta}{\beta}\right)$$

for $\frac{5}{8} \le \frac{\theta}{\beta} \le \frac{7}{8}$,

$$y = h\left(4.6660917\frac{\theta}{\beta} - 2.44406188\left(\frac{\theta}{\beta}\right)^2 - 1.2292650\right)$$

$$y' = \frac{h}{\beta}\left(4.6660928 - 4.888124\frac{\theta}{\beta}\right)$$

$$y'' = \frac{h}{\beta^2}(-4.888124)$$

$$y''' = 0$$

for $\frac{7}{8} \leq \frac{\theta}{\beta} \leq 1,$

$$y = h\left(0.6110155 + 0.3889845\frac{\theta}{\beta} - 0.0309544\sin\left(4\pi\frac{\theta}{\beta}\right)\right)$$

$$y' = \frac{h}{\beta}\left(0.3889845 - 0.3889845\cos\left(4\pi\frac{\theta}{\beta}\right)\right)$$

$$y'' = \frac{h}{\beta^2}\left(-4.888124\sin\left(4\pi\frac{\theta}{\beta}\right)\right)$$

$$y''' = 61.425975\frac{h}{\beta^3}\cos\left(4\pi\frac{\theta}{\beta}\right).$$

A computer solution is employed to establish the incremental displacement value and the characteristic curves of the action.

The modified trapezoidal curve has the following peak values

$$y' = 2\frac{h}{\beta}$$

$$y'' = 4.888\frac{h}{\beta^2}$$

$$y''' = 61.43\frac{h}{\beta^3}. \tag{3.9}$$

The nondimensional factors of the displacement, the velocity, and the acceleration of this curve are given in App. B.

EXAMPLE *A cam rotates at 300 rpm. A symmetrical modified trapezoidal acceleration curve (parabolic motion combined with the cycloidal curve) is to be drawn with the ratio $b = 1/8$. The total rise is 4 inches in 160 degrees of cam rotation. Find pertinent values of all the characteristics and plot the curves without the use of Eqs. (3.7) through (3.9).*

Solution *In Fig. 3.6 we see the basic cycloidal curve from which the combination curve is developed. This figure also shows the modified trapezoidal acceleration curve. The variables pertaining to the cycloidal sector will be denoted by the primed symbols (Fig. 3.6*a*). In Fig. 3.6*b*, let us divide one-half of the rise into its three component parts. Since $b = 1/8$ and the angle $\beta/2$ is 80 degrees, we see that the cam angle for parts 1 and 3 is $\theta_1 = \theta_3 = 20$ degrees $= \beta'/4$. This gives $\theta_2 = 40$ degrees. The angular velocity of the cam is*

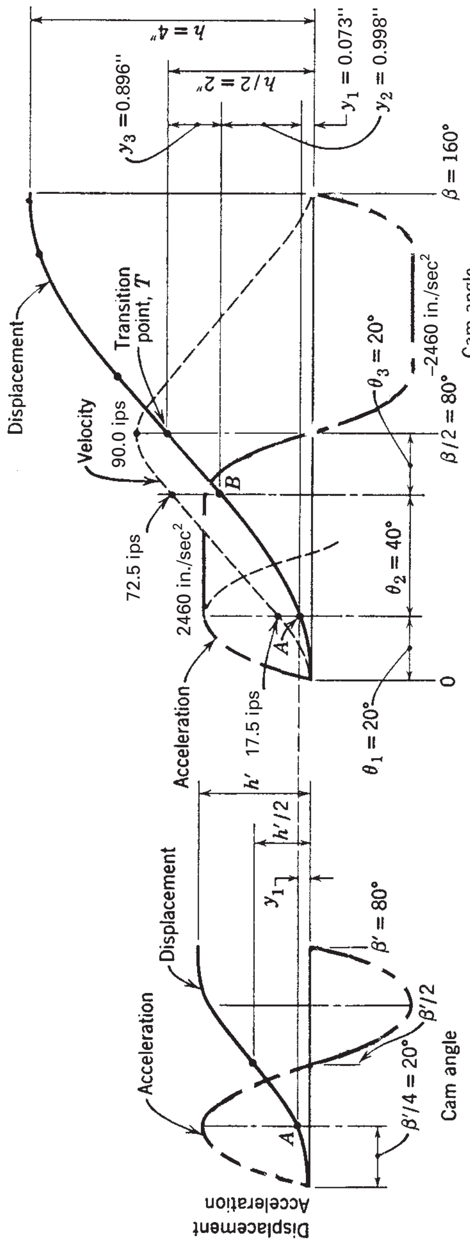
$$\omega = 300/60 \times 2\pi = 31.4 \text{ rad/sec}$$

The characteristics of the cycloidal curve from Eqs. (2.58), (2.59), and (2.60) are

$$\text{Displacement} \quad y = h\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right)\text{in}$$

$$\text{Velocity} \quad \dot{y} = \omega y' = \frac{\omega h}{\beta}\left(1 - \cos\frac{2\pi\theta}{\beta}\right)\text{ips}$$

$$\text{Acceleration} \quad \ddot{y} = \omega^2 y^1 = \left(\frac{\omega^2 2h\pi}{\beta^2}\sin\frac{2\pi\theta}{\beta}\right)\text{in/sec}^2$$



(a) Basic cycloidal curve.

(b) Modified trapezoidal curve.

FIGURE 3.6. Example modified trapezoidal acceleration curve.

At point A substituting

$$y_A = y_1 = \frac{h'}{4} - \frac{h'}{2\pi} = 0.091\,h'$$

$$\dot{y}_A = \frac{h'\omega}{\beta'} = \frac{h'(31.4)}{\frac{80}{180}\pi} = 22.5\,h'$$

$$\ddot{y}_A = A_2 = \frac{2\pi h'\omega^2}{(\beta')^2}$$

$$= \frac{2\pi h'(31.4)^2}{\left(\frac{80}{180}\pi\right)^2} = 3170\,h'$$

The displacement of part 2, the parabolic curve, in Sec. 2.6

$$y_2 = v_A\frac{\theta_2}{\omega} + \frac{1}{2}A_2\left(\frac{\theta_2}{\omega}\right)^2$$

Substituting, we obtain

$$y_2 = \frac{h'\omega}{\beta'}\frac{\theta_2}{\omega} + \frac{1}{2}\frac{2\pi h'\omega^2}{(\beta')^2}\left(\frac{\theta_2}{\omega}\right)^2$$

$$= \frac{h'\theta_2}{\beta'} + \frac{h'\pi\theta_2^2}{(\beta')^2}$$

$$= \frac{h'}{2} + h'\pi\left(\frac{1}{4}\right) = 1.285h'$$

Also, the velocity at point B

$$\dot{y}_B = v_A + A_2\frac{\theta_2}{\omega}$$

$$= 22.5h' + 3170h'\left(\frac{40\pi/180}{31.4}\right) = 93.3h'$$

The displacement of part 3

$$\dot{y}_3 = v_3\frac{\theta_3}{\omega} + \frac{h'}{2\pi} = 93.3h'\left(\frac{20\pi/180}{31.4}\right) + \frac{h'}{2\pi} = 1.199h'$$

we know that

$$y_1 + y_2 + y_3 = 2$$

Substituting,

$$0.091h' + 1.285h' + 1.199h' = 2$$
$$h' = 0.7767 \text{ in}$$

Let us now find the displacements:

$$y_1 = 0.091(0.7767) = 0.071 \text{ in}$$
$$y_2 = 1.285(0.7767) = 0.998 \text{ in}$$
$$y_3 = 1.191(0.7767) = 0.896 \text{ in}$$

Substituting to find the velocity

$$\begin{aligned} v_A &= 22.5(0.7767) = 17.5 \text{ ips} \\ v_B &= 93.3(0.7767) = 72.5 \text{ ips} \\ v_T &= \text{maximum velocity} \\ &= v_B + \Delta v_{B \text{ to } T} \\ &= 72.5 + 17.5 = 900 \text{ ips} \end{aligned}$$

Also, the maximum acceleration

$$A_A = 3170(0.7767) = 2460 \text{ in/sec}^2$$

The curves may now be plotted in Fig. 3.6*b*. If they were to be compared with a trapezoidal acceleration curve, we would find that this curve has a slightly lower maximum acceleration and the advantage of lower required cutting accuracy in the initial and final rise portions. Also, the vibrations induced at high speeds should be slightly smaller than those of the trapezoidal curve.

3.6 SKEWED MODIFIED TRAPEZOIDAL CURVE

Occasionally, the follower requires a particular velocity and acceleration at some critical points in the machine motion. This can be accomplished by skewing the acceleration profile as seen in Fig. 3.7. Neklutin (1969) has treated the modified trapezoidal curve with unequal periods of acceleration, positive and negative. Ragsdell and Gilkey (1969) have related the skewed acceleration to a correspondingly symmetrical one. Before skewing is considered, the follower rise h and angle β have been determined and will be considered as constants.

Let β_1 and β_2 be the periods during positive and negative acceleration, respectively, and let $p = \dfrac{\beta_1}{\beta_2}$ be the skew ratio (Fig. 3.7). Thus

$$\begin{aligned} \beta_1 + \beta_2 &= \beta \\ h_1 + h_2 &= h. \end{aligned}$$

The velocity match at the transition point that

$$(y'_{\max})_1 = (y'_{\max})_2$$

Substituting and equating from Eq. (3.9)

$$\frac{2h_1}{\beta_1} = \frac{2h_2}{\beta_2}$$

$$\therefore\ \beta_1 = \frac{p}{1+p}\beta \qquad (3.10)$$

$$h_1 = \frac{p}{1+p}h$$

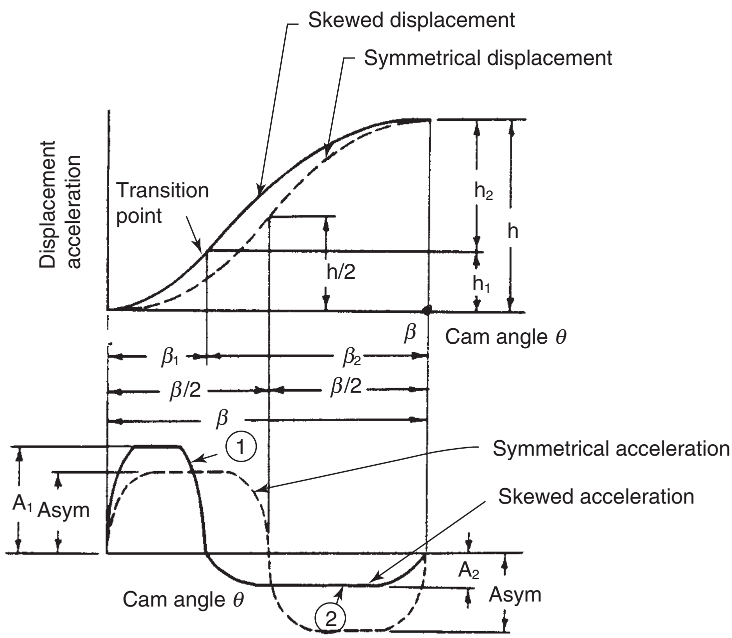


FIGURE 3.7. Skewed modified trapezoidal curve.

and

$$\beta_2 = \frac{p}{1+p}\beta$$
$$h_2 = \frac{p}{1+P}h. \tag{3.11}$$

The relationship between the skewed and corresponding symmetrical acceleration, since the maximum velocity is the same in both skewed and symmetric cases, is:

$$2\beta_1 a_1 = a_{sym}\beta = V_{max}$$
$$\therefore \quad a_1 = a_{sym}\frac{\beta}{2\beta_1}$$

Thus using Eq. (3.10)

$$a_1 = \left(\frac{1+p}{2p}\right)a_{sym}. \tag{3.12}$$

Similarly

$$a_2 = \left(\frac{1+p}{2}\right)a_{sym}. \tag{3.13}$$

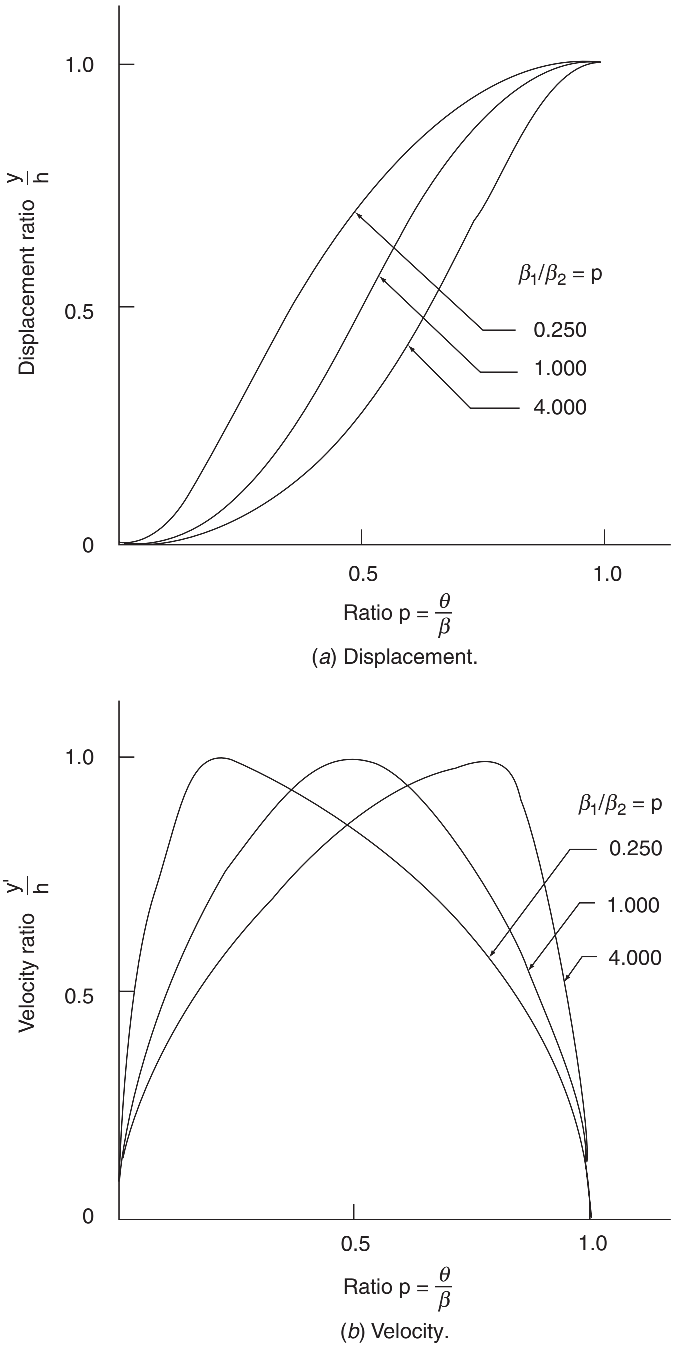


FIGURE 3.8. Skewed modified trapezoidal curve comparison (normalized $h = 1$, $\beta = 1$).

Figure 3.8*a* shows the normalized displacement plot and Fig. 3.8*b* shows the normalized velocity plot where $h = 1$, $\beta = 1$.

3.7 MODIFIED SINE CURVE

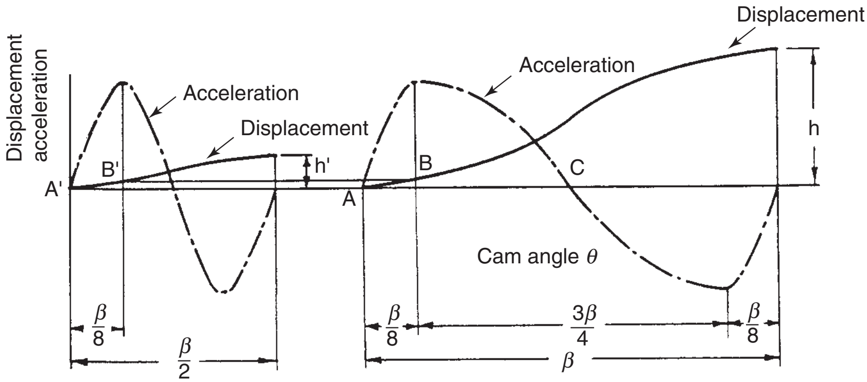
The modified sine curve (Chen, 1982; Schmidt, 1960) is a combination of quarter sine wave curves. In terms of its torsional action, the change from positive to negative torque occurs in over 40 percent of the travel time. This attribute makes this curve attractive as a choice in moving large masses such as indexing intermittent turrets. Its lower torque and power demand make the modified sine curve one of the best choices of curves.

Figure 3.9*a* shows the basic cycloidal curve from which the combination curve is developed, and Fig. 3.9*b* shows the displacement and the acceleration diagram of the modified sine curve. The primed symbols used in the drawing refer to the basic cycloidal curve. One-half of the rise is divided into the following segments; the follower is accelerated from A to B: $\left(\text{from } \theta = 0 \text{ to } \theta = \dfrac{\beta}{8}\right)$ with a quarter sine wave, and the acceleration decreased to zero from B to C $\left(\text{from } \theta = \dfrac{\beta}{8} \text{ to } \theta = \dfrac{\beta}{2}\right)$, again with a quarter sine wave. The equations of cycloidal motion from A to B, given that $\beta/8$ is the length of the initial quarter sine wave, are:

$$y = h'\left(\frac{2\theta}{\beta} - \frac{1}{2\pi}\sin 4\pi\frac{\theta}{\beta}\right)$$

$$y' = \frac{h'}{\beta}\left(2 - 2\cos 4\pi\frac{\theta}{\beta}\right)$$

$$y'' = \frac{8\pi h'}{\beta^2}\sin 4\pi\frac{\theta}{\beta}.$$



(*a*) Cycloidal curve. (*b*) Modified sine curve.

FIGURE 3.9. Modified sine curve.

At the end of the first segment $\theta = \dfrac{\beta}{8}$, and equations at point B are

$$
\begin{aligned}
y_1 &= h'\left(\frac{1}{4} - \frac{1}{2\pi}\right) \\
y_1' &= \frac{2h'}{\beta} \\
y_1'' &= \frac{8\pi h'}{\beta^2}.
\end{aligned}
\tag{3.14}
$$

The sine curve from B to C characteristics are:

$$
\begin{aligned}
y &= y_1 + C_1 + C_2 \frac{\theta - \frac{\beta}{8}}{\beta} + C_3 \sin\left(\frac{4\pi\theta}{3\beta} + \frac{\pi}{3}\right) \\
y' &= \frac{C_2}{\beta} + C_3 \frac{4\pi}{3\beta} \cos\left(\frac{4\pi\theta}{3\beta} + \frac{\pi}{3}\right) \\
y'' &= -C_3 \frac{16\pi^2}{9\beta^2} \sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)
\end{aligned}
\tag{3.15}
$$

The acceleration boundary conditions give:

$$
\frac{8\pi h'}{\beta^2} = \left[-C_3 \frac{16\pi^2}{9\beta^2} \sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)\right]_{\theta = \frac{\beta}{8}}
$$

$$
C_3 = -\frac{9h'}{2\pi}
$$

The velocity boundary conditions give:

$$
\frac{2h'}{\beta} = \left[\frac{C_2}{\beta} - \frac{9h'}{2\pi}\left(\frac{4\pi}{3\beta}\right) \cos\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)\right]_{\theta = \frac{\beta}{8}}
$$

$$
C_2 = 2h'
$$

The displacement boundary conditions give:

$$
\left[C_1 + 2h' \frac{\theta - \frac{\beta}{8}}{\beta} - \frac{9h'}{2\pi} \sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)\right]_{\theta = \frac{\beta}{8}} = 0
$$

$$
C_1 = \frac{9h'}{2\pi}
$$

Then

$$y = \left(\frac{h'}{4} - \frac{h'}{2\pi}\right) + \frac{9h'}{2\pi} + 2h'\left(\frac{\theta - \frac{\beta}{8}}{\beta}\right) - \frac{9h'}{2\pi}\sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)$$
$$= h'\left[\frac{4}{\pi} + 2\frac{\theta}{\beta} - \frac{9}{2\pi}\sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)\right]. \tag{3.16}$$

When $\theta = \frac{\beta}{2}$, the rise is

$$y = h'\left(1 + \frac{4}{\pi}\right)$$

Hence,

$$y_2 = y - y_1 = \left(\frac{3}{4} + \frac{9}{2\pi}\right)h'.$$

Finally, from the relationship

$$y = y_1 + y_2 = \frac{h}{2}$$

we obtain $h' = \frac{\pi}{2(\pi + 4)}h$.

Therefore, the displacement equations of the modified sine curve are

$$y = h\left[\frac{\pi}{4+\pi}\frac{\theta}{\beta} - \frac{1}{4(4+\pi)}\sin\left(4\pi\frac{\theta}{\beta}\right)\right] \qquad 0 \le \theta \le \frac{\beta}{8}$$
$$y = h\left[\frac{2}{4+\pi} + \frac{\pi}{4+\pi}\frac{\theta}{\beta} - \frac{9}{4(4+\pi)}\sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)\right] \qquad \frac{\beta}{8} \le \theta \le \frac{7}{8}\beta \tag{3.17}$$
$$y = h\left[\frac{4}{4+\pi} + \frac{\pi}{4+\pi}\frac{\theta}{\beta} - \frac{1}{4(4+\pi)}\sin\left(4\pi\frac{\theta}{\beta}\right)\right] \qquad \frac{7}{8}\beta \le \theta \le \beta.$$

Evaluating all constants the characteristics equations for the modified sine curve are:

for $\quad 0 \le \frac{\theta}{\beta} \le \frac{1}{8}$

$$y = h\left(0.43990\frac{\theta}{\beta} - 0.35014\sin 4\pi\frac{\theta}{\beta}\right)$$
$$y' = 0.43990\frac{h}{\beta}\left(1 - \cos 4\pi\frac{\theta}{\beta}\right)$$
$$y'' = 5.52796\frac{h}{\beta^2}\sin 4\pi\frac{\theta}{\beta}$$
$$y''' = 69.4664\frac{h}{\beta_3}\cos 4\pi\frac{\theta}{\beta}.$$

for $\frac{1}{8} \le \frac{\theta}{\beta} \le \frac{7}{8}$

$$y = h\left[0.28005 + 0.43990\frac{\theta}{\beta} - 0.351506\cos\left(\frac{4\pi}{3}\frac{\theta}{\beta} - \frac{\pi}{3}\right)\right]$$
$$y' = \frac{h}{\beta}\left[0.43990 + 1.31970\sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)\right]$$
$$y'' = 5.52796\frac{h}{\beta^2}\sin\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{3}\right)$$
$$y''' = 23.1555\frac{h}{\beta^3}\cos\left(\frac{4\pi}{3}\frac{\theta}{\beta} + \frac{\pi}{4}\right).$$

for $\frac{7}{8} \le \frac{\theta}{\beta} \le 1$

$$y = h\left[0.56010 + 0.43990\frac{\theta}{\beta} - 0.03515006\sin 4\pi\frac{\theta}{\beta}\right]$$
$$y' = \frac{h}{\beta}\left[0.43989\left(1 - \cos 4\pi\frac{\theta}{\beta}\right)\right]$$
$$y'' = 5.52796\frac{h}{\beta^2}\sin 4\pi\frac{\theta}{\beta} \tag{3.18}$$
$$y''' = 69.4664\frac{h}{\beta^3}\cos 4\pi\frac{\theta}{\beta}.$$

A computer solution is employed to establish the incremental displacement values and the characteristic curves of the action. The maximum velocity of the modified sine curve is $y'_{\max} = 1.760\frac{h}{\beta}$, the maximum acceleration is $y''_{\max} = 5.528\frac{h}{\beta^2}$, and the maximum jerk is $y'''_{\max} = 69.47\frac{h}{\beta^3}$. The nondimensionalized displacement, velocity, and acceleration factors are given in Table A-4, App. B. Figure 3.10 indicates the comparison (Erdman and Sandor, 1997) of the cycloidal, modified trapezoidal, and modified sine curves. The data shown is for a 3-inch pitch diameter cam having a 2-inch rise in 6 degrees of cam rotation.

3.8 MODIFIED CYCLOIDAL CURVE

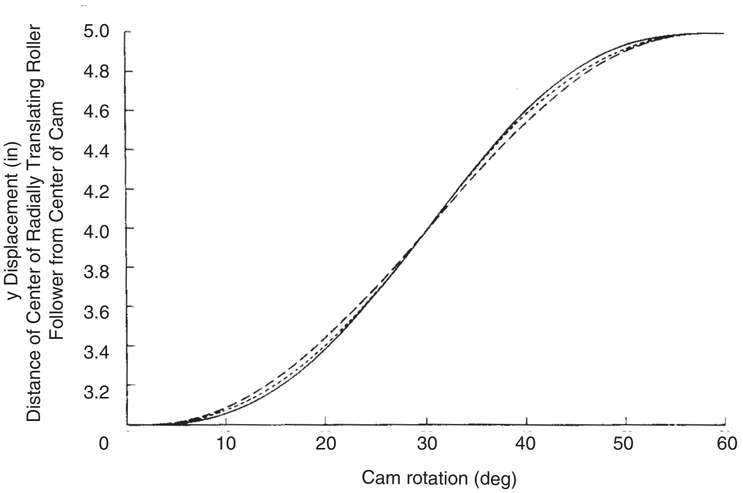
In this section we will reshape the cycloidal curve to improve its acceleration characteristics. This curve is the modified cycloidal curve that was developed by Wildt (1953). Figure 3.11 indicates the acceleration comparison between the true cycloidal curve and the Wildt cycloidal curve. The basic cycloidal curve equation for the displacement

$$y = h\left(\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\frac{2\pi\theta}{\beta}\right) \tag{2.58}$$

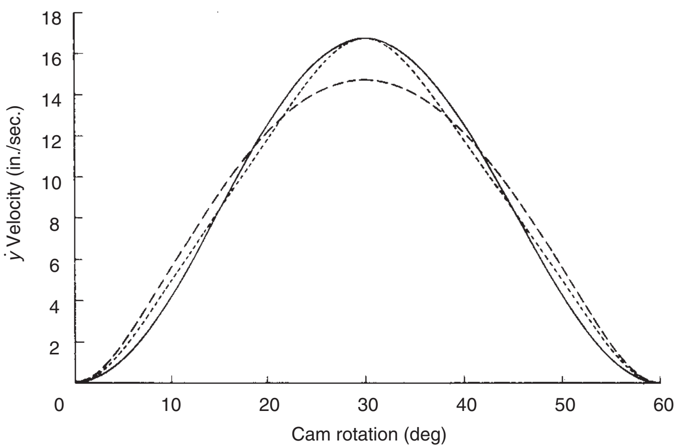
From this equation it is seen that the cycloidal curve is a combination of a sine curve and a constant velocity line. Figure 3.12a shows the pure cycloidal curve with point A

the beginning of motion, point B the end of motion, and point P the mid-stroke transition point. APB is the constant velocity line. M is the midpoint between A and P. The sine amplitudes are added to the constant-velocity line in this true cycloidal case.

The modified cycloidal curve was developed to maximize the orientation of the superimposed sine wave amplitude on the straight line; see geometric construction in Fig. 3.12*b*. In this figure, a point D equal to 0.57 the distance $\frac{\beta}{2}$ is the first chosen and then is joined to M by a straight line. The base of the sine curve is then constructed

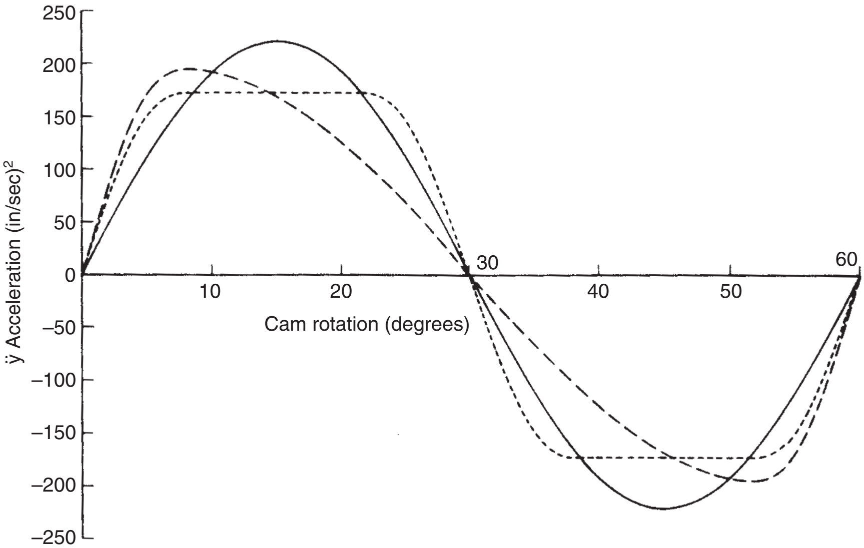


(*a*) Displacement.



(*b*) Velocity.

FIGURE 3.10. Comparison of popular curves (cam has 2-inch rise in 60-degree rotation and 3-inch pitch diameter). (Erdman and Sandor (1997) with permission by Prentice Hall, Upper Saddle River, N.J.)



(*c*) Acceleration.

FIGURE 3.10. *Continued*

perpendicular to *DM*. This procedure results in the modified cycloidal curve having a maximum acceleration about 7 percent lower than that of the basic cycloidal curve.

3.9 DWELL-RISE-RETURN-DWELL MOTION

Now, let us consider the dwell-rise-return-dwell curve, using a combination of curves to improve the high-speed action. To analyze the action, we shall use the symmetrical cycloidal curve (Fig. 3.13), although any of the high-speed shapes, such as trapezoidal and modified trapezoidal, introduce the same problem. A difficulty arises that did not prevail in the dwell-rise-dwell action, that is, an abrupt change or dip in the acceleration curve occurs at the maximum rise point. This dip is undesirable, because it produces sudden inertia loads and vibrations.

In Fig. 3.13, the problem is eliminated by blending a parabolic curve to the cycloidal curve portions. A modified total curve is produced. Also, a smoother, more desirable acceleration curve is developed with lower peak accelerations. An alternative solution, necessitating less mathematical work, is to employ polynomial equations as shown in Chap. 4.

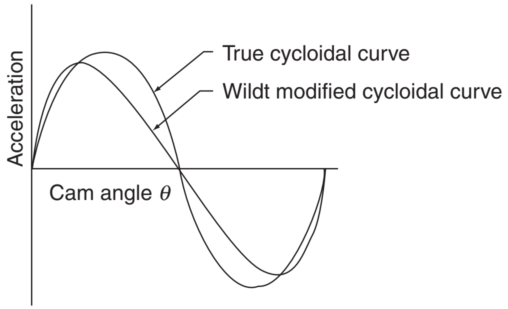
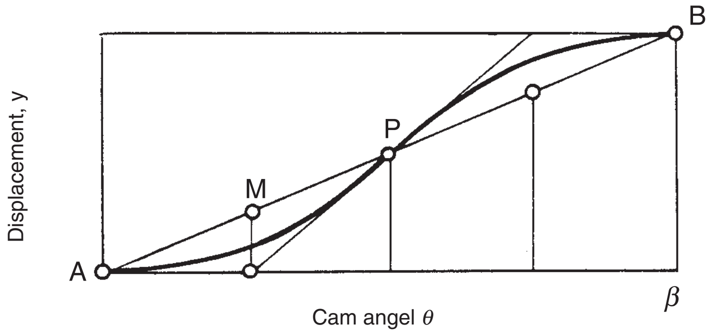
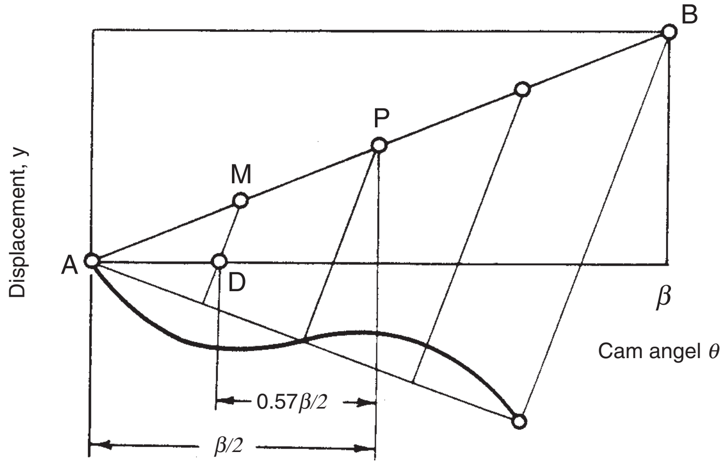


FIGURE 3.11. Comparison of cycloidal acceleration curves.



(*a*) Pure cycloidal curve.



(*b*) Modified cycloidal curve.

FIGURE 3.12. Modified cycloidal curve construction.

EXAMPLE *Derive the relationship for the dwell-rise-fall-dwell cam curve shown in Fig. 3.14 having equal maximum acceleration values. Portions I and III are harmonic; portions II and IV are horizontal straight lines.*

Solution *Let the θ's and β's be the angles for each portion shown. Note that for velocity and acceleration one should multiply values by ω and ω^2, respectively, and the boundary conditions are $y(0) = 0$, $\dot{y}(0) = 0$, $\dot{y}(\beta_4) = 0$, and $y(\beta_4)$ = total rise h. Use basic trigonometric relationships for*

Portion I

$$\ddot{y} = A\sin(\pi\theta/2\beta_1)$$
$$\dot{y} = (2\beta_1 A/\pi)[1-\cos(\pi\theta/2\beta_1)]$$
$$y = -A(2\beta_1/\pi)^2\sin(\pi\theta/2\beta_1) + (2A\beta_1/\pi)\theta$$
$$\dot{y}_1 = 2A\beta_1/\pi$$
$$y_1 = (2A\beta_1^2/\pi)(1-2/\pi)$$

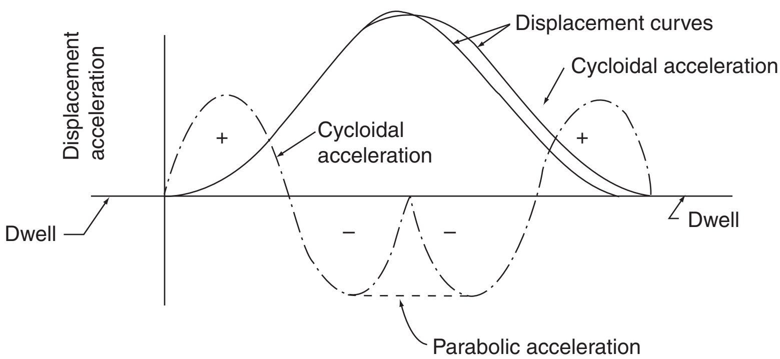


FIGURE 3.13. Dwell-rise-return-dwell curves, symmetrical rise-fall.

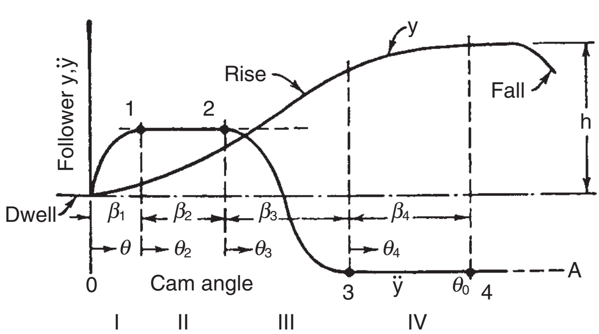


FIGURE 3.14. Example of asymmetrical dwell-rise-fall-dwell cam curve.

Portion II

$$\ddot{y} = A$$
$$\dot{y} = A\theta_2 + \dot{y}_1$$
$$y = A(\theta_2)^2/2 + \dot{y}_1\theta' + y_1$$
$$\dot{y}_2 = A\beta_2 + 2A\beta_1/\pi$$
$$y_2 = A\beta_2^2/2 + 2A\beta_1\beta_2/\pi + 2A\beta_1^2/\pi(1 - 2/\pi)$$

Portion III

$$\ddot{y} = A\ \cos(\pi\theta_3/\beta_3)$$
$$\dot{y} = A\beta_3/\pi \sin(\pi\theta_3/\beta_3) + \dot{y}_2$$
$$y = -A(\beta_3/\pi)^2[1 - \cos(\pi\theta''/\beta_3)] + \dot{y}_2\theta'' + y_2$$
$$\dot{y}_3 = \dot{y}_2$$
$$y_3 = 2A(\beta_3/\pi)^2 + A\beta_2\beta_3 + 2A\beta_1\beta_3/\pi + A\beta_2^2/2 + 2A\beta_1\beta_2/\pi + (2A\beta_1^2/\pi)(1 - 2/\pi)$$

Portion IV

$$\ddot{y} = -A$$
$$\dot{y} = -A\theta_4 + \dot{y}_3$$
$$y = -\left[A(\theta_4)^2/2\right] + \dot{y}_3\theta''' + y_3$$
$$\dot{y}_4 = -A\beta_2 + \dot{y}_3 = -A\beta_4 + A\beta_2 + 2A\beta_1/\pi = 0$$
$$\beta_4 = \beta_2 + 2\beta_1/\pi$$

Total Rise

$$h = y_4 = -(A\beta_4^2/2) + (A\beta_2 + 2A\beta_1/\pi)\beta_4 + y_3$$

Substituting

$$A = \frac{h}{\begin{array}{c}[-\beta_4^2/2 + \beta_2\beta_4 + 2\beta_1\beta_4/\pi + 2(\beta_3/\pi)^2 + \beta_2\beta_3 + 2\beta_1\beta_3/\pi + \beta_2^2/2 + 2\beta_1\beta_2/\pi \\ +(2\beta_1^2/\pi)(1 - 2/\pi)]\end{array}}$$

For a given total rise h for angle θ_0 and any two of the β angles given one can solve for all angles and all values of the derivative curves.

3.10 COUPLED CURVE SIMPLIFICATION

This section presents a convenient method for combining segments of basic curves to produce the required design motion. This procedure was developed by Kloomok and Muffley in Mabie and Ocvirk (1979), who selected three analytic functions of the simple harmonic, the cycloidal, and the eighth-degree polynomial (the latter described in Chapter 4). These curves, having excellent characteristics, can be blended with constant acceleration, constant velocity, and any other curve satisfying the boundary conditions stated in Sec. 3.2. Figures 3.15, 3.16, and 3.17 show the three curves including both half curve segments and full curve action in which

h = total follower displacement for half curve or full curve action, and
β = cam angle for displacement h, in

The displacement, velocity, and acceleration characteristics of the curves are indicated. The simple harmonic curve (Fig. 3.15) has a low maximum acceleration for a given rise. Its acceleration discontinuity at the ends of the DRD cycle can be matched with the cycloidal segments (Fig. 3.16) to produce the popular cycloidal coupling. The cycloidal segments are excellent choices to eliminate the acceleration curve discontinuities of any DRD curve by its blending segment. The eighth polynomial, Fig. 3.17, is a good choice

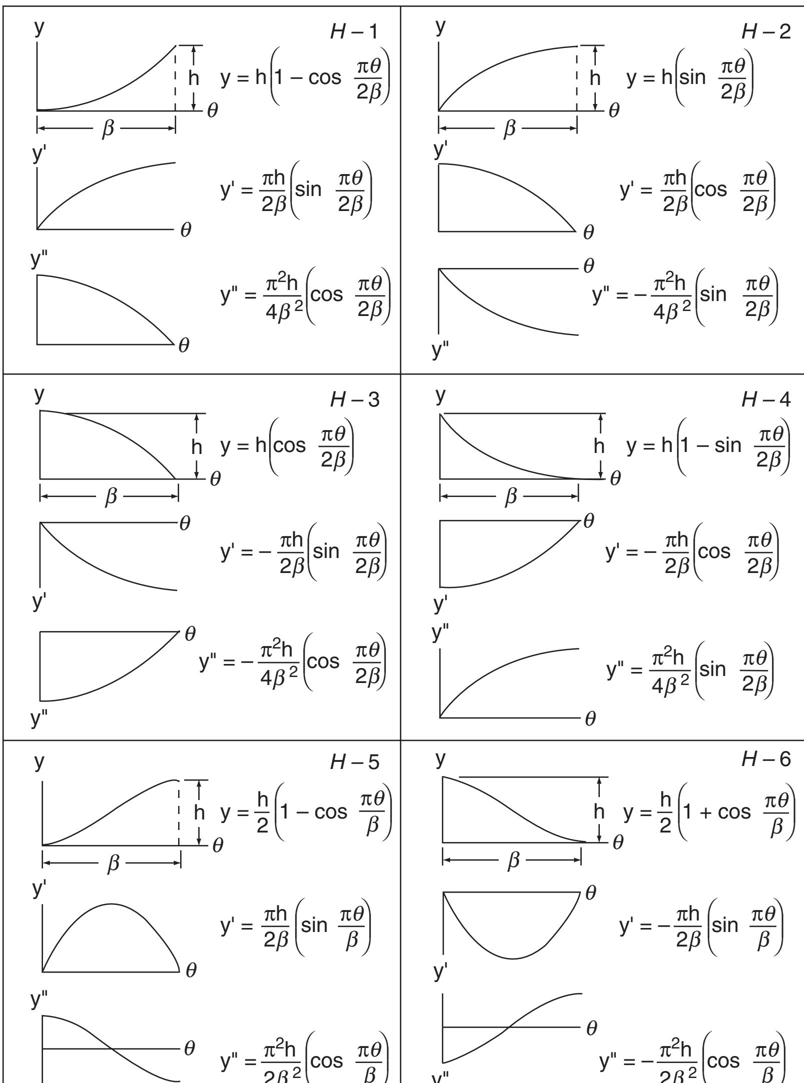


FIGURE 3.15. Harmonic motion characteristics.
Note: h = total follower displacement for half-curve or full-curve action, and β = cam angle for displacement h, in.

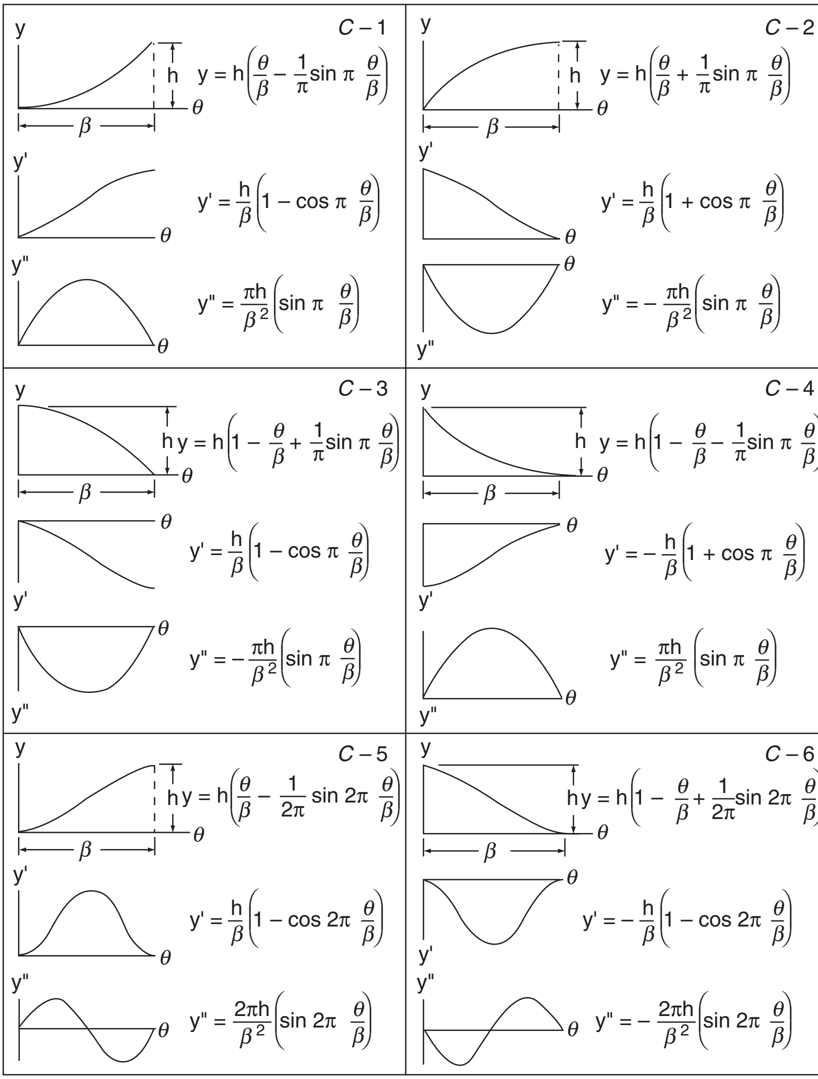
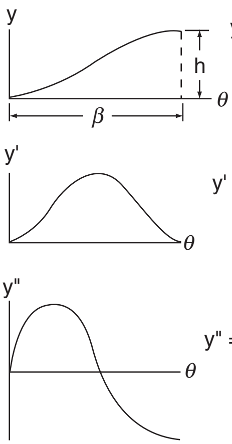


FIGURE 3.16. Cycloidal motion characteristics.
Note: h = total follower displacement for half-curve or full curve action, and β = cam angle for displacement h.

for blending with curves in general, especially with DRRD cam action. Chapter 4 shows an elaborate treatment of polynomials.

EXAMPLE *A spring-loaded textile machine has a cam that rotates at 720 rpm in which the follower rises $1\frac{3}{4}$ inches in 120 degrees. To keep the spring size small, the maximum positive acceleration is twice the maximum negative acceleration. Design (a) the rise portion of the DRRD system with harmonic and cycloidal coupling using the simplified*

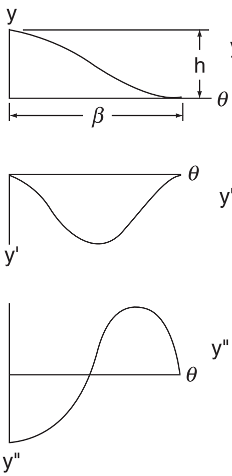


$$y = h\left[6.09755\left(\frac{\theta}{\beta}\right)^3 - 20.78040\left(\frac{\theta}{\beta}\right)^5 + 26.73155\left(\frac{\theta}{\beta}\right)^6 - 13.60965\left(\frac{\theta}{\beta}\right)^7 + 2.56095\left(\frac{\theta}{\beta}\right)^8\right]$$

$$y' = \frac{h}{\beta}\left[18.29265\left(\frac{\theta}{\beta}\right)^2 - 103.90200\left(\frac{\theta}{\beta}\right)^4 + 160.38930\left(\frac{\theta}{\beta}\right)^5 - 95.26755\left(\frac{\theta}{\beta}\right)^6 + 20.48760\left(\frac{\theta}{\beta}\right)^7\right]$$

$$y'' = \frac{h}{\beta^2}\left[36.58530\left(\frac{\theta}{\beta}\right) - 415.60800\left(\frac{\theta}{\beta}\right)^3 + 801.94650\left(\frac{\theta}{\beta}\right)^4 - 571.60530\left(\frac{\theta}{\beta}\right)^5 + 143.41320\left(\frac{\theta}{\beta}\right)^6\right]$$

P–1



$$y = h\left[1.00000 - 2.63415\left(\frac{\theta}{\beta}\right)^2 + 2.78055\left(\frac{\theta}{\beta}\right)^5 + 3.17060\left(\frac{\theta}{\beta}\right)^6 - 6.87795\left(\frac{\theta}{\beta}\right)^7 + 2.56095\left(\frac{\theta}{\beta}\right)^8\right]$$

$$y' = \frac{h}{\beta}\left[-5.26830\left(\frac{\theta}{\beta}\right) + 13.90275\left(\frac{\theta}{\beta}\right)^4 + 19.02360\left(\frac{\theta}{\beta}\right)^5 - 43.14565\left(\frac{\theta}{\beta}\right)^6 + 20.48760\left(\frac{\theta}{\beta}\right)^7\right]$$

$$y'' = \frac{h}{\beta^2}\left[-5.26830 + 55.61100\left(\frac{\theta}{\beta}\right)^3 + 95.11800\left(\frac{\theta}{\beta}\right)^4 - 288.87390\left(\frac{\theta}{\beta}\right)^5 + 143.41320\left(\frac{\theta}{\beta}\right)^6\right]$$

P–2

FIGURE 3.17. Eighth-degree polynomial motion characteristics.
Note: h = total follower displacement.
β = cam angle for displacement h, in.

method of this section. Also find (b) at point the characteristic values of displacement, velocity, and acceleration.

Solution (a)
Angular velocity

$$\omega = \frac{720 \times 2\pi}{60} = 75.398 \text{ rad/sec}$$
$$h = 1.75 \text{ in.}$$
$$\beta = \frac{(120)2\pi}{360} = 2.094 \text{ rad}$$

From Fig. 3.16 for cycloidal motion (C1 segment), the characteristic curves are

$$y = h_1\left(\frac{\theta}{\beta} - \frac{1}{\pi}\sin\frac{\pi\theta}{\beta_1}\right)$$

$$y' = \frac{h_1}{\beta_1}\left(1 - \cos\frac{\pi\theta}{\beta_1}\right)$$

$$y'' = \frac{\pi h_1}{(\beta_1)^2}\sin\frac{\pi\theta}{\beta_1}$$

From Fig. 3.15 for harmonic motion (H2 segment) the characteristic curves are

$$y = h_2 \sin\frac{\pi\theta}{2\beta_2}$$

$$y' = \frac{\pi h_2}{2\beta_2}\cos\frac{\pi\theta}{2\beta_2}$$

$$y'' = -\frac{\pi^2 h_2}{4(\beta_2)^2}\sin\frac{\pi\theta}{2\beta_2}$$

We need velocity continuity at the end of the C1 curve and the beginning of the H2 curve (Fig. 3.18)

$$\frac{h_1}{\beta_1}\left(1 - \cos\frac{\pi\beta_1}{\beta_1}\right) = \frac{\pi h_2}{2\beta_2}\cos\frac{\pi 0}{2\beta_2}$$

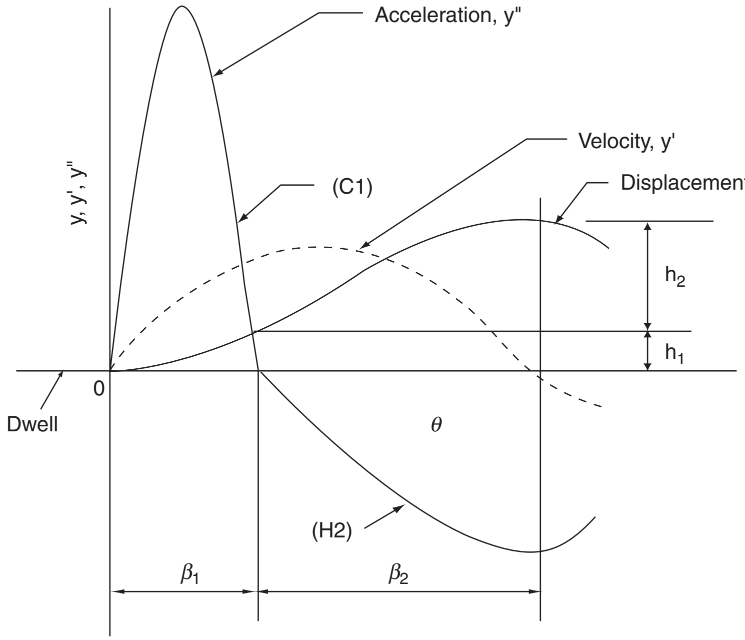


FIGURE 3.18. Cycloidal harmonic compling example.

which yields

$$\frac{4h_1}{\pi h_2} = \frac{\beta_1}{\beta_2}.$$

For the C1 curve the acceleration is

$$y''_{\max} = \frac{\pi h_1}{\beta_1^2}$$

And for the H2 curve, the acceleration is

$$y''_{\max} = \frac{\pi^2 h_2}{4\beta_2^2}$$

It is given that $y''_{\max}$ for C1 curve = 2 × $y''_{\max}$ for H2 curves. Therefore

$$\frac{\pi h_1}{\beta_1^2} = \left| -\frac{\pi^2 h_2}{2\beta_2^2} \right|$$

Now we have four equations to solve for h_1, h_2, β_1, β_2.

$$h_1 + h_2 = 1.75 \text{ in}$$
$$\beta_1 + \beta_2 = 2.094 \text{ rad}$$
$$\frac{4h_1}{\pi h_2} = \frac{\beta_1}{\beta_2}$$
$$\left| \frac{\pi h_2}{\beta_1^2} \right| = \left| -\frac{\pi^2 h_2}{2\beta_2^2} \right|$$

This solves to yield

$$h_1 = 0.4934 \text{ in}$$
$$h_2 = 1.2566 \text{ in}$$
$$\beta_1 = 40 \text{ deg} \times \frac{\pi}{180} = 0.6981 \text{ rad}$$
$$\beta_2 = 80 \text{ deg} \times \frac{\pi}{180} = 1.3963 \text{ rad}$$

By following the above principles, the designer may complete the DRRD curve (utilizing H3 and C4) and modify it to suit the design conditions of the machine.

Solution (b)
The 15 degrees falls in the cycloidal curve C1 region

$$15 \text{ deg} = \frac{15\pi}{180} = 0.262 \text{ rad}$$

The displacement

$$y = 0.493\left[\frac{0.262}{0698} - \frac{1}{\pi}\sin\frac{\pi(0.262)}{0.698}\right] = 0.400 \text{ in}$$

The velocity

$$y' = \frac{0.493}{0.698}\left[1 - \cos\frac{0.262\pi}{0.698}\right] = 0.437 \text{ in}$$

$$\dot{y} = \omega y' = 75.398(0.437) = 32.93 \text{ in/sec}$$

The acceleration

$$y'' = \frac{0.493\pi}{(0.698)^2}\left(\sin\frac{0.262\pi}{0.698}\right) = 2.938 \text{ in/rad}^2$$

$$\dot{y} = \omega^2 y'' = (75.398)^2(2.938) = 6700 \text{ in/sec}^2$$

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