VUMAT for Fabric Reinforced Composites

1. Introduction

This document describes a constitutive model for fabric reinforced composites that was introduced in Abaqus/Explicit 6.8. The model has been implemented as a built-in VUMAT user subroutine. It can be accessed by naming your material such that it begins with the string `ABQ_PLY_FABRIC`, e.g. `ABQ_PLY_FABRIC_1`.

The model is currently supported for plane-stress elements; this includes shell (S4R and S3R), continuum shell (SC6R and SC8R), plane stress (CPS family) and membrane (M3D family) elements. User materials are currently not supported in ABAQUS/Explicit for small-strain shell elements (S4RS).

When user-defined materials are employed to define the material response of shell elements, ABAQUS/Explicit cannot calculate a default value for the transverse shear stiffness of the element. Hence, you must manually define the element's transverse shear stiffness. See “Shell section behavior,” Section 24.6.4 of the Version 6.8 ABAQUS Analysis User’s Manual, for guidelines on choosing this stiffness.

This document describes the basic equations of the constitutive model and provides detailed information of the user interface to the VUMAT implementation in ABAQUS.

2. Continuum damage model for fabric reinforced composites

A schematic representation of the geometry of the woven fabric reinforcement considered in the constitutive model is shown in Figure 1. The fiber directions are assumed to be orthogonal.

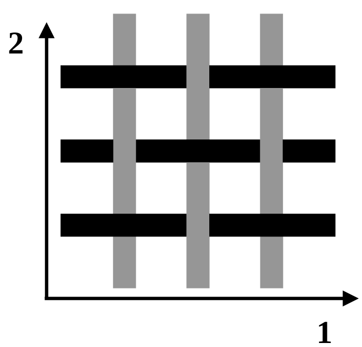

Figure 1: *Schematic representation of woven fabric. Fibers are aligned with directions 1 and 2 of a local coordinate system.*

The constitutive stress-strain relations are formulated in a local Cartesian coordinate system with base vectors aligned with the fiber directions, as shown in Figure 1.

The fabric-reinforced ply is modeled as a homogeneous orthotropic elastic material with the potential to sustain progressive stiffness degradation due to fiber/matrix cracking, and plastic deformation under shear loading. The different aspects of the model are discussed next.

2.1. Elastic stress-strain relations

It is assumed that the elastic stress-strain relations are given by orthotropic damaged elasticity. Referred to a local coordinate system aligned with the fiber directions (Figure 1) the elastic relations take the form:

$$\begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12}^{el} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{(1-d_1)E_1} & \dfrac{-\nu_{12}}{E_1} & 0 \\ \dfrac{-\nu_{21}}{E_2} & \dfrac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \dfrac{1}{(1-d_{12})2G_{12}} \end{pmatrix} \begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{pmatrix} \tag{1}$$

The damage variables d_1 and d_2 are associated with fiber fracture along the 1 and 2 directions respectively, whereas d_{12} is related to matrix micro-cracking due to shear deformation. The model differentiates between tensile and compressive fiber failure modes by activating the corresponding damage variable depending on the stress state in the fiber directions. Thus:

$$d_1 = d_{1+} \frac{<\sigma_{11}>}{|\sigma_{11}|} + d_{1-} \frac{<-\sigma_{11}>}{|\sigma_{11}|}; \quad d_2 = d_{2+} \frac{<\sigma_{22}>}{|\sigma_{22}|} + d_{2-} \frac{<-\sigma_{22}>}{|\sigma_{22}|} \tag{2}$$

In order to incorporate different initial (undamaged) stiffness in tension and compression, the values of the elastic constants E_1, E_2 and ν_{12} are assumed to take their tensile or compressive values depending on the sign of $tr(\boldsymbol{\varepsilon}) = \varepsilon_{11} + \varepsilon_{22}$.

2.2. Fiber response

The material response along the fiber directions is characterized with damaged elasticity. It is assumed that the fiber damage variables are a function of the corresponding effective stress, that is:

$$\begin{aligned} d_{1+} &= d_{1+}(\tilde{\sigma}_{1+}) \qquad & d_{1-} &= d_{1-}(\tilde{\sigma}_{1-}) \\ d_{2+} &= d_{2+}(\tilde{\sigma}_{2+}) \qquad & d_{2-} &= d_{2-}(\tilde{\sigma}_{2-}) \end{aligned} \tag{3}$$

The effective stresses are defined as:

$$\begin{aligned} \tilde{\sigma}_{1+} &= \frac{<\sigma_{11}>}{(1-d_{1+})} \qquad & \tilde{\sigma}_{1-} &= \frac{<-\sigma_{11}>}{(1-d_{1-})} \\ \tilde{\sigma}_{2+} &= \frac{<\sigma_{22}>}{(1-d_{2+})} \qquad & \tilde{\sigma}_{2-} &= \frac{<-\sigma_{22}>}{(1-d_{2-})} \end{aligned} \tag{4}$$

In order to simplify notation, an index α will be used in subsequent discussions, such that it takes the value $\alpha = 1(+/-), 2(+/-)$, depending on the sign of the corresponding stresses. Thus, the above four equations for the damage variables are rewritten as:

$$d_\alpha = d_\alpha(\tilde{\sigma}_\alpha) \tag{5}$$

It is noted that the effective stresses $\tilde{\sigma}_\alpha$ are directly related to the thermodynamic forces, Y_α, that are work conjugate to the damage variables, through the relationship $\tilde{\sigma}_\alpha = \sqrt{2E_\alpha Y_\alpha}$. Therefore the above equation states that the fiber damage variables depend only on the corresponding thermodynamic force.

At any given time the elastic domain is defined in terms of the damage activation functions, F_α, as

$$F_\alpha = \phi_\alpha - r_\alpha \le 0 \tag{6}$$

The functions ϕ_α provide a criterion for fiber failure and are assumed to take the form

$$\phi_\alpha = \frac{\tilde{\sigma}_\alpha}{X_\alpha}; \qquad (\alpha = 1+, 1-, 2+, 2-) \tag{7}$$

where X_α are the tensile/compressive strengths for uniaxial loading along the fiber directions.

The damage thresholds, r_α, are initially set to one. After damage activation ($\phi_\alpha = 1$) they increase with increasing damage according to:

$$r_\alpha(t) = \max_{\tau \le t} \phi_\alpha(\tau) \tag{8}$$

The definition ensures that the damage thresholds are non-decreasing quantities ($\dot{r}_\alpha(t) \ge 0$). The damage thresholds are assumed to obey the Kuhn-Tucker complementary conditions:

$$F_\alpha \le 0 \qquad \dot{r}_\alpha \ge 0 \qquad \dot{r}_\alpha F_\alpha = 0 \tag{9}$$

and the consistency condition:

$$\dot{r}_\alpha \dot{F}_\alpha = 0 \tag{10}$$

Note that the formulation can be easily enhanced to take the effects of damage upon load reversal into account. For example, compressive damage will usually degrade the tensile response if the loading is reversed from compression to tension. On the other hand, tensile cracks close under compressive loading and have little effect on the compressive response.

The evolution of the damage variables are a function of the damage thresholds and the fracture energy per unit area under uniaxial tensile/compressive loading, G_f^α. The formulation of the damage evolution law ensures that the damage variables are monotonically increasing quantities. It also ensures that the correct amount of energy is dissipated when the lamina is subjected to uniaxial loading conditions along the fiber directions.

The evolution of the damage variables is given by the equation:

$$d_\alpha = 1 - \frac{1}{r_\alpha} \exp\left(-A_\alpha (r_\alpha - 1)\right); \qquad \dot{d}_\alpha \ge 0\,, \tag{11}$$

where

$$A_\alpha = \frac{2 g_0^\alpha L_c}{G_f^\alpha - g_0^\alpha L_c} \tag{12}$$

Here, L_c is the characteristic length of the element, G_f^{α} is the fracture energy per unit area under uniaxial tensile/compressive loading, and g_0^{α} is the elastic energy density (i.e. per unit volume) at the point of damage initiation:

$$g_0^{\alpha} = \frac{X_{\alpha}^2}{2E_{\alpha}} \tag{13}$$

The formulation of the damage evolution law ensures that the damage variables are monotonically increasing quantities. It also ensures that the correct amount of energy is dissipated when the lamina is subjected to uniaxial loading conditions along the fiber directions. For instance, under uniaxial tensile loading in the fiber 1 direction, the dissipated energy per unit area is equal to the fracture energy G_f^{1+}.

This holds true provided that:

$$G_f^{\alpha} - g_0^{\alpha} L_c > 0 \Leftrightarrow L_c < L_{\max} = \frac{G_f^{\alpha}}{g_0^{\alpha}} \tag{14}$$

The formulation therefore imposes a restriction on the maximum element size that can be used to accurately capture the right amount of energy dissipation during fracture. If the characteristic element size of the FE mesh is greater than $L_{\max}$, the analysis will over-predict the energy dissipation.

Note that, from Abaqus 6.10-EF1 onwards, the critical element length for each of the ABQ_PLY_FABRIC materials and a representative list of elements that exceed this criterion are printed in the `.sta` file.

2.3. Shear response

As mentioned earlier, the shear response is dominated by the non-linear behavior of the matrix, which includes both plasticity and stiffness degradation due to matrix microcracking. The main ingredients of the shear response are discussed below.

2.3.1 Elasticity

The elastic relations give the effective (undamaged) stress in terms of elastic strain:

$$\tilde{\sigma}_{12} = \frac{\sigma_{12}}{(1-d_{12})} = 2G_{12}\varepsilon_{12}^{el} = 2G_{12}(\varepsilon_{12} - \varepsilon_{12}^{pl}) \tag{15}$$

2.3.2 Plasticity

- Yield function:

$$F = |\,\tilde{\sigma}_{12}\,| - \tilde{\sigma}_0(\bar{\varepsilon}^{pl}) \le 0 \tag{16}$$

The hardening function is assumed to be of the form:

$$\tilde{\sigma}_0(\bar{\varepsilon}^{pl}) = \tilde{\sigma}_{y0} + C(\bar{\varepsilon}^{pl})^p \tag{17}$$

- Flow rule: Assuming associated flow, then

$$\dot{\varepsilon}_{12}^{pl} = \dot{\bar{\varepsilon}}^{pl}\frac{\partial F}{\partial \tilde{\sigma}_{12}} = \dot{\bar{\varepsilon}}^{pl}\, sign(\tilde{\sigma}_{12}) \tag{18}$$

The evolution of the plastic work during yielding is given as

$$\dot{U}^{pl} = \sigma_{12}\dot{\varepsilon}_{12}^{pl} = (1-d_{12})\tilde{\sigma}_{12}\dot{\varepsilon}_{12}^{pl} = (1-d_{12})\tilde{\sigma}_0\dot{\bar{\varepsilon}}^{pl} \tag{19}$$

2.3.3 Damage

The elastic domain is defined in terms of the damage activation function, F_{12}, as

$$F_{12} = \phi_{12} - r_{12} \le 0 \tag{20}$$

The function ϕ_{12} provides the criterion for initiation of shear damage of the matrix, which is assumed to be of the form

$$\phi_{12} = \frac{\tilde{\sigma}_{12}}{S} \tag{21}$$

Here $\tilde{\sigma}_{12} = \sigma_{12}/(1-d_{12})$ is the effective shear stress, and S is the shear stress for initiation of matrix damage.

The damage threshold, r_{12}, is initially set to one and increases after damage activation ($\phi_{12} = 1$) according to

$$r_{12}(t) = \max_{\tau \le t} \phi_{12}(\tau) \tag{22}$$

Finally, based on [1] it is assumed that the shear damage variable increases with the logarithm of r_{12} until a maximum value $d_{12}^{\max}$ is reached. Thus:

$$d_{12} = \min(\alpha_{12}\ln(r_{12}), d_{12}^{\max}) \tag{23}$$

where $\alpha_{12} > 0$, and $d_{12}^{\max} \le 1$ are material properties.

2.4. Element Deletion

The VUMAT provides two options to delete elements:

1) The element is deleted when any one tensile/compressive damage variable along the fiber directions reaches a maximum specified value, $d_1 = d_{\max}$ or $d_2 = d_{\max}$, or when the plastic strain due to shear deformation reaches a maximum specified value, $\bar{\varepsilon}^{pl} = \bar{\varepsilon}^{pl}_{\max}$. This option is activated by setting the flag `lDelFlag=1`.

2) The element is deleted when the damage variables along both fiber directions reach a maximum specified value, $d_1 = d_2 = d_{\max}$, or when the plastic strain due to shear deformation reaches a maximum specified value, $\bar{\varepsilon}^{pl} = \bar{\varepsilon}^{pl}_{\max}$. This option is activated by setting the flag `lDelFlag=2`.

These two options can be combined with a deformation-based element deletion criterion based on the values of the maximum ($\hat{\varepsilon}_{\max} > 0$) and minimum ($\hat{\varepsilon}_{\min} < 0$) principal logarithmic strains that the element can sustain before it gets deleted.

3. Calibration procedure

The elastic constants and the fiber tension/compression strengths, X_α, are easily measured from standard coupon tests in uniaxial tension/compression loading of 0/90 laminates. The calibration of damage evolution in the fiber failure modes is based on the fracture energy per unit area of the material, G_f^α, which can be measured experimentally.

The shear response is usually calibrated with a cyclic tensile test on a ±45 laminate, where the strains along the fiber directions can be neglected. Figure 2 shows the typical shear response of a fabric reinforced composite. It is noted that the unloading/reloading paths in this figure correspond to an idealization of the actual response, which usually exhibits hysteretic behavior. The figure will serve as the starting point for the discussion of a general calibration procedure for the parameters that enter the damage and plasticity equations.

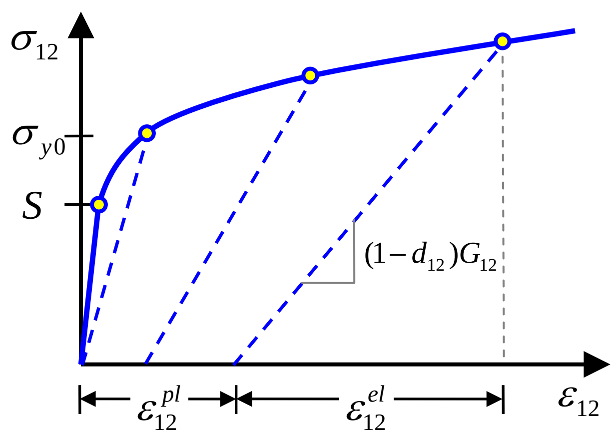

Figure 2: *Schematic representation of typical shear response of a fabric reinforced composite.*

The level of damage can be measured from the ratio of the unloading stiffness to the initial (undamaged) elastic stiffness. This allows us to compute pairs of stress-damage values, (σ_{12}, d_{12}), for each unloading curve. This data can be represented in the space of d_{12} versus $\ln(\tilde{\sigma}_{12})$, where $\tilde{\sigma}_{12} = \sigma_{12}/(1-d_{12})$. A linear fit of the data provides the values of α_{12} (slope of the line) and S (intersection with the horizontal axis) as shown in Figure 3. Sometimes the damage data shows indication of a saturation value, which would be used to determine $d_{12}^{\max} \le 1$. Otherwise a value of $d_{12}^{\max} = 1$ should be used.

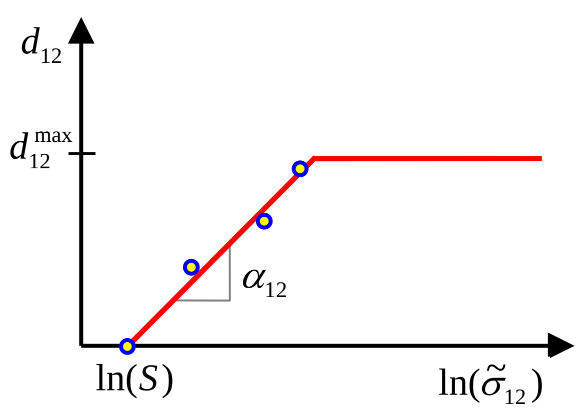

Figure 3: *Calibration of the shear damage parameters α_{12} and S*

Finally, for each unloading curve in Figure 2, the plastic strain ε_{12}^{pl} at the onset of unloading is determined from the value of residual deformation in the unloaded state. The values of $(\tilde{\sigma}_{12}, \varepsilon_{12}^{pl})$ at the onset of unloading are then used to fit the parameters of the hardening curve, as illustrated in Figure 4.

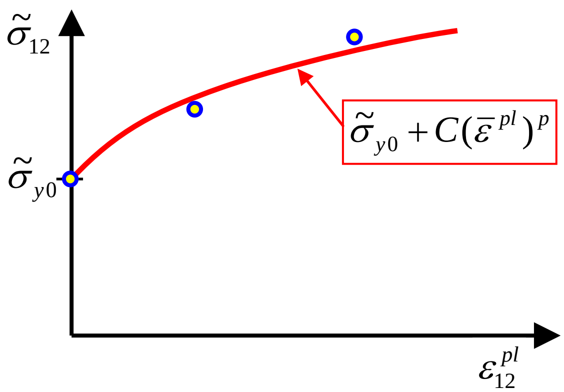

Figure 4: *Calibration of the shear hardening curve*

4. User interface

To activate the model for fabric reinforced composites the material name must start with the string `ABQ_PLY_FABRIC`, e.g. `ABQ_PLY_FABRIC_1`. A synopsis of the interface is shown below. The number of solution dependent variables (under keyword option `*DEPVAR`) is 16, and the `DELETE` parameter is equal to 16. Refer to Table 1 for a detailed description of each material constant specified in the keyword interface.

```
*MATERIAL, NAME= ABQ_PLY_FABRIC
*DENSITY
 ρ
*USER MATERIAL, CONSTANTS=40
** Line 1:
 E_1+, E_2+, ν_12+, G_12, E_1-, E_2-, ν_12-
** Line 2:
 X_1+, X_1-, X_2+, X_2-, S
** Line 3:
 G_f^1+, G_f^1-, G_f^2+, G_f^2-, α_12, d_12^max
** Line 4:
 σ̃_y0, C, p
** Line 5:
 lDelFlag, d_max, ε̄_max^pl, ε̂_max, ε̂_min
*DEPVAR, DELETE=16
 16
```

Table 1*: User material constants for the fabric reinforced composite material model.*

LINE: 1

Pos.	Symbol	Description
1	E_{1+}	Young's modulus along fiber direction 1 when $tr(\boldsymbol{\varepsilon}) \geq 0$
2	E_{2+}	Young's modulus along fiber direction 2 when $tr(\boldsymbol{\varepsilon}) \geq 0$
3	ν_{12+}	Poisson ratio when $tr(\boldsymbol{\varepsilon}) \geq 0$
4	G_{12}	Shear modulus
5	E_{1-}	Young's modulus along fiber direction 1 when $tr(\boldsymbol{\varepsilon}) < 0$
6	E_{2-}	Young's modulus along fiber direction 2 when $tr(\boldsymbol{\varepsilon}) < 0$
7	ν_{12-}	Poisson ratio when $tr(\boldsymbol{\varepsilon}) < 0$
8		*Not used*

LINE: 2 Damage initiation coefficients

Pos.	Symbol	Description
1	X_{1+}	Tensile strength along fiber direction 1
2	X_{1-}	Compressive strength along fiber direction 1
3	X_{2+}	Tensile strength along fiber direction 2
4	X_{2-}	Compressive strength along fiber direction 2
5	S	Shear stress at the onset of shear damage
6-8		*Not used*

LINE: 3 Damage evolution coefficients

Pos.	Symbol	Description
1	G_f^{1+}	Energy per unit area for tensile fracture along fiber direction 1
2	G_f^{1-}	Energy per unit area for compressive fracture along fiber direction 1
3	G_f^{2+}	Energy per unit area for tensile fracture along fiber direction 2
4	G_f^{2-}	Energy per unit area for compressive fracture along fiber direction 2
5	α_{12}	Parameter in the equation of shear damage
6	$d_{12}^{\max}$	Maximum shear damage
7-8		*Not used*

LINE: 4 Shear plasticity coefficients: $\tilde{\sigma}_0(\bar{\varepsilon}^{pl}) = \tilde{\sigma}_{y0} + C(\bar{\varepsilon}^{pl})^p$

Pos.	Symbol	Description
1	$\tilde{\sigma}_{y0}$	Initial effective shear yield stress
2	C	Coefficient in hardening equation
3	p	Power term in hardening equation
4-8		*Not used*

LINE: 5 Controls for material point failure

Pos.	Symbol	Description
1	`lDelFlag`	Element deletion flag: `lDelFlag=0:` Element is not deleted (default) `lDelFlag=1:` Element is deleted when either fiber fails, $d_1 = d_{\max}$ or $d_2 = d_{\max}$, or when $\bar{\varepsilon}^{pl} = \bar{\varepsilon}^{pl}_{\max}$ `lDelFlag=2:` Element is deleted when both fibers fail, $d_1 = d_2 = d_{\max}$, or when $\bar{\varepsilon}^{pl} = \bar{\varepsilon}^{pl}_{\max}$.
2	$d_{\max}$	Maximum value of damage variable used in element deletion criterion
3	$\bar{\varepsilon}^{pl}_{\max}$	Maximum value of equivalent plastic strain for element deletion criterion. (A value of zero means that $\bar{\varepsilon}^{pl}$ is not used as criterion for element deletion)
4	$\hat{\varepsilon}_{\max}$	Maximum (positive) principal logarithmic strain beyond which the element will get deleted. Ignored if zero, not specified, or `lDelFlag=0`.
5	$\hat{\varepsilon}_{\min}$	Minimum (negative) principal logarithmic strain beyond which the element will get deleted. Ignored if zero, not specified, or `lDelFlag=0`.
6-8		*Not used*

5. Output

In addition to the standard (material-independent) output variables in Abaqus/Explicit for stress-displacement elements (such as stress, S, strain, LE, element STATUS, etc.) the following output variables have a special meaning for the user material for fabric-reinforced composites:

Output Variable	Symbol	Description
SDV1	d_{1+}	Tensile damage along fiber direction 1
SDV2	d_{1-}	Compressive damage along fiber direction 1
SDV3	d_{2+}	Tensile damage along fiber direction 2
SDV4	d_{2-}	Compressive damage along fiber direction 2
SDV5	d_{12}	Shear damage
SDV6	r_{1+}	Tensile damage threshold along fiber direction 1
SDV7	r_{1-}	Compressive damage threshold along fiber direction 1
SDV8	r_{2+}	Tensile damage threshold along fiber direction 2
SDV9	r_{2-}	Compressive damage threshold along fiber direction 2
SDV10	r_{12}	Shear damage threshold
SDV11	$\bar{\varepsilon}^{pl}$	Equivalent plastic strain
SDV12	ε_{11}^{el}	Elastic strain component 11
SDV13	ε_{22}^{el}	Elastic strain component 22
SDV14		Not used
SDV15	ε_{12}^{el}	Elastic strain component 12
SDV16	MpStatus	Material point status: 1 if active, 0 if failed.

6. References

[1] Alastair F. Johnson and Josef Simon, “Modeling Fabric Reinforced Composites under Impact Loads”. In *EUROMECH 400: Impact and Damage Tolerance of Composite Materials and Structures*. Imperial College of Science Technology & Medicine, London 27-29 September 1999.