A Practical Analysis of Slackline Forces

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Abstract

We attempt to analyze the tensions in a strand of elastic material strung horizontally between two anchors, with a load applied somewhere in the middle. First we analyze the tensions caused by a non-moving load, which can be derived purely from the geometry of the system. Next, we derive a system of ordinary differential equations to describe the motion in the line when a load is applied to it with a specified initial position and velocity. Initially, our equations make no assumptions about the linearity of the elastic material. Under a linearity assumption, however, the differential equations can be solved explicitly, and several significant quantities can be calculated using only the geometry of the system. In the last sections, we analyze some data gathered from these calculations.

1 Introduction

1.1 Motivation

In a typical slackline set-up, a length of tubular nylon webbing (an elastic material) is strung horizontally between two anchors, which we will assume are at the same height relative to gravity. The line is stretched to a certain initial tension (which is more or less impossible to calculate without taking

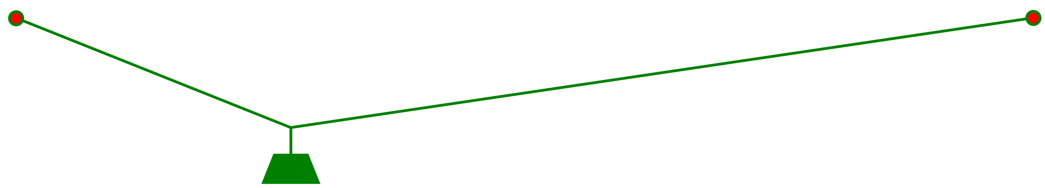

Figure 1: Diagram of a slackline set-up

direct measurements), after which people attempt to stand, walk, jump, and do various feats of acrobatics on the line. In the first section of this article, we will treat a person standing on the line as a static load, and calculate the tension in the line based solely on the person's weight and the geometry of the system.

When slacklining is done high above the ground, the person usually wears a safety leash, which runs from a harness around their waist to a small metal ring (often a climbing carabiner) around the slackline itself. As they walk, the ring around the line is dragged along behind them, and if they fall off the line, this safety leash will catch them. This means that the person's weight will be applied to the line dynamically, as they will fall a short distance before the safety leash comes tight on the line. It is widely believed that applying this sort of dynamic load to the line will generate significantly higher tensions in the line, and thus in particular will place significantly higher forces on the anchors at each end of the line. In order to understand how much force these anchors must be able to support, it is important to know the magnitude of the tensions that can be generated in a slackline in various situations, including dynamic ones.

Note that in the case of a fall as described above, it is extremely important that the slackline is elastic, so that it can stretch to absorb the impact of the fall. If the line had very low elasticity (e.g. if steel cable were used instead of nylon webbing), then even if some slack were left in the line, then unless the safety leash itself could stretch significantly to absorb the impact, the fall would be arrested by a very sudden jolting stop. This could cause serious bodily harm to the person who fell, and could also put tremendous forces on the anchors.

DISCLAIMER: Slacklining anywhere other than at ground level is inherently dangerous and may result in severe injury or death. Do not attempt this before seeking proper instruction from a qualified professional. Furthermore, while the author of this document can be reasonably certain of the correctness of the mathematics contained herein, he makes no claims whatsoever as to the relevance of his conclusions to anything in the real world. Do not use the information presented here to conclude that a certain slackline set-up is safe.

1.2 Notation

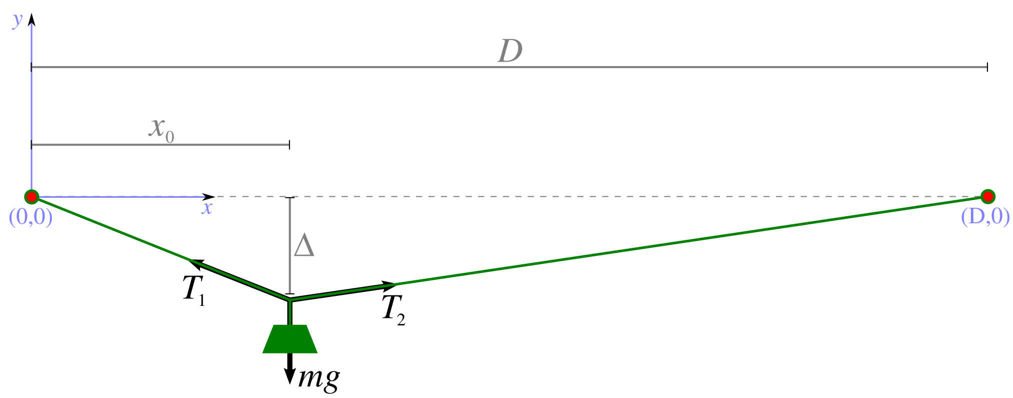

Figure 2: Force Diagram

Variables:

t = time
$(x(t), y(t))$ = position of load relative to the left anchor (origin)

Constants:

$$
\begin{aligned}
L &= \text{original unstretched length of webbing} \\
D &= \text{horizontal distance between anchors} \\
m &= \text{mass of the load} \\
g &= \text{gravitational acceleration} \\
\Delta &= \text{how far down the load pulls the line when } \textit{not moving} \\
&\ \text{(i.e. the } \textit{depression} \text{ of the line under a static load)} \\
x_0 &= \text{initial horizontal position (in dynamic case)} \\
y_0 &= \text{initial vertical position (in dynamic case)} \\
v_0 &= \text{initial speed (in dynamic case)}
\end{aligned}
$$

The last two items will vary from one scenario to another, but will be constant throughout any given scenario.

Unknown:

$$F(\lambda) = \text{tension in the line when it has stretched by a factor of } \lambda$$

We will assume that the webbing stretches in a uniform (i.e. linear) way along its entire length. (This has nothing to do with the tension $F(\lambda)$ being linear as a function of λ, and should be a perfectly reasonable assumption.) We will also assume that the load is initially applied at $(x_0, 0)$ when the line is straight and horizontal, and that it never slides left or right along the webbing as the webbing moves. (That point on the webbing may still move left or right as it moves up and down, but we are assuming the load is fixed to that point and does not slide.) In other words, it is as if the webbing were cut at the point where the load is applied, and both of the now separate segments to the left and to the right were affixed to the load. To simplify the terminology, we will call the webbing to the left of the load the "left segment" and the webbing to the right of the load the "right segment". This implies the following:

Unstretched length of left segment: $\frac{x_0}{D}L$

Unstretched length of right segment: $(1 - \frac{x_0}{D})L$

Stretched length of left segment: $\sqrt{x^2+y^2}$

Stretched length of right segment: $\sqrt{(D-x)^2+y^2}$

Tension in left segment: $T_1 = F\left(\frac{\sqrt{x^2+y^2}}{\frac{x_0}{D}L}\right)$

Tension in right segment: $T_2 = F\left(\frac{\sqrt{(D-x)^2+y^2}}{(1-\frac{x_0}{D})L}\right)$

Direction vector for T_1: $\left(\frac{-x}{\sqrt{x^2+y^2}}, \frac{-y}{\sqrt{x^2+y^2}}\right)$

Direction vector for T_2: $\left(\frac{D-x}{\sqrt{(D-x)^2+y^2}}, \frac{-y}{\sqrt{(D-x)^2+y^2}}\right)$

Note that the above expressions for the tensions T_1 and T_2 obey a certain symmetry: replacing x_0 with $D - x_0$ in the expression for T_1 will give the expression for T_2, and vice versa. This is due to the symmetry in the geometry of the system: the tension in the left segment when the load is x_0 meters from the left anchor should be exactly the same as the tension in the right segment when the load is x_0 meters from the right anchor, i.e. $D-x_0$ meters from the left anchor. This same symmetry will hold in all of the expressions that we will derive for T_1 and T_2, so in what follows, we will show fewer details about the expressions for T_2.

2 Preliminary Analysis

2.1 Static Load Analysis

If the load is not moving, then the sum of the three force vectors on the load (tensions T_1 and T_2, and gravity) is zero. This gives us the following two equations:

$$\begin{cases} T_1 \cdot \frac{-x}{\sqrt{x^2+y^2}} + T_2 \cdot \frac{D-x}{\sqrt{(D-x)^2+y^2}} = 0 \\ T_1 \cdot \frac{-y}{\sqrt{x^2+y^2}} + T_2 \cdot \frac{-y}{\sqrt{(D-x)^2+y^2}} - mg = 0 \end{cases} \tag{1}$$

Solving these simultaneously gives

$$\begin{aligned} T_1 &= mg\left(1-\frac{x}{D}\right)\sqrt{\left(\frac{x}{y}\right)^2+1} \\ T_2 &= mg\left(\frac{x}{D}\right)\sqrt{\left(\frac{D-x}{y}\right)^2+1} \end{aligned}$$

These solutions have no predictive value at all, because without knowing more about $F(\lambda)$ for our material, we can't possibly predict the tensions in the line. In particular, for given values of D and x and the load mg, we have cannot predict what y will be. However, these solutions do allow us to calculate the tensions in a real world system in which we can *measure* y along with D, x and mg. To rephrase that, we can calculate the tensions in the line using nothing more than the geometry of the system and the weight of the load. (This may seem counterintuitive at first, because it seems at first glance that the tension should depend on how tight the line is pulled. It does, but indirectly, in the sense that the tighter the line is pulled, the smaller y will be, and hence the greater the values of the expressions above will be.) Note that, by the notation introduced in section 1.2 and Figure 2, this value of y is $-\Delta$. Also, note that when the load is in the middle of the line, i.e. when $x = \frac{D}{2}$, the tensions will be equal, and the expressions above simplify to

$$T_1 = T_2 = \frac{1}{4}mg\sqrt{\left(\frac{D}{y}\right)^2+4}$$

2.2 Differential Equations

If we allow for a moving load, then by Newton's second law, equation (1) becomes the following system of differential equations:

$$\begin{cases} F\left(\frac{\sqrt{x^2+y^2}}{\frac{x_0}{D}L}\right)\frac{-x}{\sqrt{x^2+y^2}} + F\left(\frac{\sqrt{(D-x)^2+y^2}}{\left(1-\frac{x_0}{D}\right)L}\right)\frac{D-x}{\sqrt{(D-x)^2+y^2}} = mx'' \\ F\left(\frac{\sqrt{x^2+y^2}}{\frac{x_0}{D}L}\right)\frac{-y}{\sqrt{x^2+y^2}} + F\left(\frac{\sqrt{(D-x)^2+y^2}}{\left(1-\frac{x_0}{D}\right)L}\right)\frac{-y}{\sqrt{(D-x)^2+y^2}} - mg = my'' \end{cases} \tag{2}$$

Here we have replaced T_1 and T_2 by the expressions derived earlier for them in terms of the function $F(\lambda)$. Note that in this form, the equations are a coupled system of second order ordinary differential equations, with the single independent variable t. If, at some point, enough data on the behavior of the function F becomes known, then it would be an easy matter to numerically solve this system of equations for various values of the parameters and appropriate initial conditions. This could potentially yield much more accurate predictions than the results presented here.

However, at this point we must make a lovely observation. If we assume that F is a linear function, that is that $F(\lambda) = K\lambda$ for some constant K (measured in units of force), then (2) simplifies to

$$\begin{cases} -\frac{KD}{Lx_0}x + \frac{KD}{L(D-x_0)}(D-x) = mx'' \\ \\ -\frac{KD}{Lx_0}y - \frac{KD}{L(D-x_0)}y - mg = my'' \end{cases} \tag{3}$$

In this form, the equations have magically uncoupled, and each of the now separate differential equations is linear (nonhomogeneous) with constant coefficients! Furthermore, in all the cases we will consider, the initial conditions for x will be $x(0) = x_0$, $x'(0) = 0$, and it is immediately clear that these conditions yield the constant solution $x(t) = x_0$ for the first equation.

Therefore, in all that follows, we will concern ourselves solely with the second of these differential equations, which we write in standard form as

$$y'' + \frac{K}{Lm\frac{x_0}{D}\left(1 - \frac{x_0}{D}\right)}y = -g \tag{4}$$

This is the equation of a simple undamped harmonic oscillator. As is standard practice with such an equation, we now define

$$\omega = \sqrt{\frac{K}{Lm\frac{x_0}{D}\left(1 - \frac{x_0}{D}\right)}}$$

Then (4) becomes

$$y'' + \omega^2 y = -g$$

and its general solution is

$$y = C_1 \cos(\omega t) + C_2 \sin(\omega t) - \frac{g}{\omega^2} \tag{5}$$

Note that the constant solution of this (when C_1 and C_2 are both 0) is $y = -\frac{g}{\omega^2}$. That is, this is the value of y when there is no movement in the line at all. We have previously called this value $-\Delta$. So we now have

$$\Delta = \frac{g}{\omega^2} = L \cdot \frac{mg}{K} \cdot \frac{x_0}{D}\left(1 - \frac{x_0}{D}\right) \tag{6}$$

Recall that the conclusion of Section 2.1 was that although we could not predict this value, it is extremely useful, for the case of a static load in a real world setting, to *measure* this value. It will turn out that, even though we are now dealing with a dynamic load rather than a static one, this quantity will play an extremely important role. In fact, in all the results that follow, every quantity that matters to us can be expressed in terms of Δ, other parameters related to the geometry of the system, and the weight of the load (mg), thus eliminating any need to know K or L, which are more difficult to measure. As an example of this, we can now rewrite the expressions for T_1 and T_2 derived in section 1.2:

$$\begin{aligned} T_1 &= F\left(\frac{\sqrt{x^2+y^2}}{\frac{x_0}{D}L}\right) \\ &= K \cdot \frac{\sqrt{x^2+y^2}}{\frac{x_0}{D}L} \\ &= mg\left(1 - \frac{x_0}{D}\right)\frac{\sqrt{x^2+y^2}}{L \cdot \frac{mg}{K} \cdot \frac{x_0}{D}\left(1 - \frac{x_0}{D}\right)} \\ &= mg\left(1 - \frac{x_0}{D}\right)\frac{\sqrt{x^2+y^2}}{\Delta} \end{aligned} \tag{7}$$

$$T_2 = mg\frac{x_0}{D}\frac{\sqrt{x^2+y^2}}{\Delta} \tag{8}$$

Finally, note in (5) that $y(0) = C_1 - \Delta$ and $y'(0) = C_2\omega$. If we use the initial conditions $y(0) = y_0$, $y'(0) = -v_0$, and solve for the constants C_1 and C_2, then our general solution takes the form

$$y = (y_0 + \Delta)\cos(\omega t) - \frac{v_0}{\omega}\sin(\omega t) - \Delta \tag{9}$$

We will be interested in two quantities related to this solution: the maximum tensions in the line, and the maximum force that acts on the load (or more specifically, the maximum "g force", i.e. acceleration, felt by the load). For the first, note that the tensions in the line increase with y^2, so to find the maximum tension, we will want to find the maximum value of y^2, which will occur when y is at its absolute minimum. Since the amplitude of the oscillator is $\sqrt{(y_0+\Delta)^2+\left(\frac{v_0}{\omega}\right)^2}$, this minimum value of y will be

$$y_{min} = -\sqrt{(y_0+\Delta)^2+\left(\frac{v_0}{\omega}\right)^2} - \Delta.$$

The maximum force on the load will occur when the upward acceleration of the load is the greatest (which also happens to be when y is at its absolute minimum.) Since the acceleration of the load is

$$y'' = -(y_0+\Delta)\omega^2\cos(\omega t) + v_0\omega\sin(\omega t)$$

and the amplitude of this oscillator is $\omega^2\sqrt{(y_0+\Delta)^2+\left(\frac{v_0}{\omega}\right)^2}$ (which simplifies to $\omega^2(|y_{min}|-\Delta)$), it follows that the maximum acceleration felt by the load is simply this amplitude plus g:

$$a_{max} = \omega^2(|y_{min}|-\Delta) + g = \left(\frac{\omega^2}{g}(|y_{min}|-\Delta)+1\right)g = \frac{|y_{min}|}{\Delta}g$$

3 Modeling a Leash Fall

It is simple to verify that if a body, acted on by constant gravitational acceleration g, is released from rest and falls a distance h, then its final speed will be $\sqrt{2gh}$. Hence if a load is dropped this distance onto our tensioned line, the speed at which it is moving when it hits the line, i.e. its initial speed v_0 in the context of section 2.2, will be $\sqrt{2gh}$. We will consider two possible cases of this.

3.1 First Model

One way to model a leash fall on a slackline is as follows. Initially, a person is standing on the line and is not moving. At this moment we have $y = -\Delta$. The person then falls off the line, and free-falls until their safety leash is fully extended and begins to pull on the line, thus applying the person's weight to the line at an initial speed equal to the speed at which the person is falling. But, as anyone who has fallen off a slackline will tell you, as soon as you fall off, the line snaps back up very quickly. Thus it may not be unrealistic to assume that by the time the person's safety leash comes tight on the line, the line itself has returned to horizontal.

This means that the distance of the free-fall was only $l - \Delta$ (where l is the length of the leash plus the length of the person's legs), and that the vertical position of the line when the load is applied is $y = 0$. Hence our initial conditions (section 2.2) should be $y_0 = 0$, $v_0 = \sqrt{2g(l - \Delta)}$. This gives us

$$y_{min} = -\sqrt{\Delta^2 + \frac{2g(l-\Delta)}{\omega^2}} - \Delta = -\sqrt{2l\Delta - \Delta^2} - \Delta$$

(using the fact that $\Delta = \frac{g}{\omega^2}$ (6)). Finally, using (7) and (8), the maximum tensions in the line are as follows:

$$\begin{aligned} T_1 &= mg\left(1 - \frac{x_0}{D}\right)\frac{\sqrt{x_0^2 + y_{min}^2}}{\Delta} \\ &= mg\left(1 - \frac{x_0}{D}\right)\frac{\sqrt{x_0^2 + 2l\Delta + 2\Delta\sqrt{2l\Delta - \Delta^2}}}{\Delta} \\ &= mg\left(1 - \frac{x_0}{D}\right)\sqrt{\left(\frac{x_0}{\Delta}\right)^2 + 2\frac{l}{\Delta} + 2\sqrt{2\frac{l}{\Delta} - 1}} \end{aligned}$$

$$T_2 = mg \cdot \frac{x_0}{D}\sqrt{\left(\frac{D - x_0}{\Delta}\right)^2 + 2\frac{l}{\Delta} + 2\sqrt{2\frac{l}{\Delta} - 1}}$$

The maximum acceleration that the person falling will feel is

$$a_{max} = \frac{|y_{min}|}{\Delta}g = \left(\sqrt{2\frac{l}{\Delta} - 1} + 1\right)g$$

3.2 Second Model

The major assumption in our first model is that the line returns all the way to horizontal before the safety leash begins to pull it back down. This makes the model a sort of "best case" scenario, because the distance of the free-fall is shortened, and the line begins absorbing the energy of the fall from a higher point. Thus the expressions derived in the previous section should be considered a lower bound for the tensions involved. To obtain an upper bound, we will consider an unrealistic, but clearly worst-case, scenario. We will assume in this model that when the person falls from the slackline, the line does not snap back up at all, but rather stays in place as if held there by some unseen force.

Thus the distance of the free-fall will be exactly l, the length of the leash plus the length of the person's legs, and the vertical position of the line when the load is applied will be the same as before the person falls, namely $y = -\Delta$. Hence our initial conditions should be $y_0 = -\Delta$, $v_0 = \sqrt{2gl}$. This gives us

$$y_{min} = -\sqrt{\frac{2gl}{\omega^2}} - \Delta = -\sqrt{2l\Delta} - \Delta$$

Thus the maximum tensions in the line in this model are as follows:

$$\begin{aligned} T_1 &= mg\left(1 - \frac{x_0}{D}\right)\frac{\sqrt{x_0^2 + y_{min}^2}}{\Delta} \\ &= mg\left(1 - \frac{x_0}{D}\right)\frac{\sqrt{x_0^2 + 2l\Delta + 2\Delta\sqrt{2l\Delta} + \Delta^2}}{\Delta} \\ &= mg\left(1 - \frac{x_0}{D}\right)\sqrt{\left(\frac{x_0}{\Delta}\right)^2 + 2\frac{l}{\Delta} + 2\sqrt{2\frac{l}{\Delta}} + 1} \end{aligned}$$

$$T_2 = mg \cdot \frac{x_0}{D}\sqrt{\left(\frac{D - x_0}{\Delta}\right)^2 + 2\frac{l}{\Delta} + 2\sqrt{2\frac{l}{\Delta}} + 1}$$

The maximum acceleration that the person falling will feel is

$$a_{max} = \frac{|y_{min}|}{\Delta}g = \left(\sqrt{2\frac{l}{\Delta}} + 1\right)g$$

3.3 Discussion

The final form of all the expressions derived in this section have involved only the weight mg of the load (person on the slackline), a few geometric measurements (D, x_0, and l), and our magical quantity Δ, the distance that the line sags with a non-moving load (at horizontal position x_0). Unfortunately, right now, it is more or less impossible to predict the value of Δ for any given slackline set-up or any position x_0 on the line. However, since we can measure Δ directly, these calculations still may have tremendous practical significance. For example, if I am walking on a slackline that spans 55 feet, and it sags 5 feet in the middle under my weight, then I can compute that the tension in either side of the line is roughly 2.8 times my body weight (section 2.1). Recall that this result is based purely on geometry and simple physics, and is more or less irrefutable.

Now, suppose this slackline is high above the ground (perhaps at the rim of Yosemite Valley), and I fall off and am caught by a leash that is 5 feet long. Since I am about 6 feet tall, my waist was roughly 3 feet above the line before the fall, and is now 5 feet below it, so $l = 8$ in this scenario. The results of this section imply that the maximum tension in the line during the fall is between 3.01 and 3.08 times my body weight, that the maximum distance below the anchors that my feet will reach is somewhere between 20.4 and 21.9 feet, and that I feel between 2.5 and 2.8 g's at the most. Unfortunately, all of these results are based on a major assumption: that the tension in the slackline is a linear function of the factor by which it has stretched. (This was the assumption that $F(\lambda) = K\lambda$ for some K in section 2.2. It more or less says that the webbing obeys Hooke's Law, or that its Young's modulus is constant.) But, as anyone who has experience setting up slacklines probably knows, this assumption is false for tubular nylon webbing. For one thing, the elastic properties of webbing are known to change as it ages, or when it gets wet, or when it is exposed to ultraviolet light for long periods of time. But even disregarding these factors, it is common to set up a slackline and find that it sags a certain distance at first, and then a few minutes later find that it sags more under the same load and needs to be pulled tighter. This implies that its Young's modulus decreases when it is kept under fairly high tension.

However, before we throw away most of the results of this paper, consider this.

All of the situations just mentioned involve the Young's modulus changing over a period of minutes or months or years. But as the example above demonstrates, the results gathered here are intended to be used by roughly measuring the amount that the line sags (Δ) when a person is just standing on it, and using this value to calculate the various other quantities that would arise if that person took a leash fall at that instant. It may not be too unrealistic to assume that, during the split second in which a fall occurs, the Young's modulus does remain more or less constant, and thus that the models developed here are valid. On the other hand, this may be completely untrue, because it is generally believed that a leash fall generates substantially higher tensions in a slackline than a static load generates, and it may be that this sort of sudden increase in tension makes the nonlinearity apparent in the webbing. We saw in the example above (and we will see more in the graphs in the next section) that if the models developed here are valid, then the tensions generated during a leash fall are not actually so high (except perhaps when they occur very close to one end of the line), so that argument may not hold. Admittedly, this is a bit of circular reasoning, but it does seem to provide some evidence that this model may be valid, at least for falls that occur in the middle of the line (where we know that the tensions on each side of the line are equal). Furthermore, it is worth mentioning that what we have done here is one version of a standard trick in the analysis of nonlinear dynamical systems. We have found an equilibrium (the position of the line when not moving) and we have linearized our differential equations about that equilibrium.

4 Results

Since all of our expressions for the two tensions T_1 and T_2 obey the symmetry described in section 1.2, we need only present data for one of these tensions. We have chosen to include only T_1 in this summary and the graphs below.

4.1 Summary of Formulas

Notation:
$D =$ Horiz. dist. b/w anchors
$x_0 =$ Horiz. dist. of person from anchor
$l =$ Length of leash + length of legs
$\Delta =$ Depression in line when person is not moving

This first formula makes no assumption about the webbing.

Tension when not moving: $mg\left(1-\frac{x_0}{D}\right)\sqrt{\left(\frac{x_0}{\Delta}\right)^2+1}$

The rest of these assume the webbing is linearly elastic.

Max. tension caused by a leash fall:

Lower bound: $mg\left(1-\frac{x_0}{D}\right)\sqrt{\left(\frac{x_0}{\Delta}\right)^2+2\frac{l}{\Delta}+2\sqrt{2\frac{l}{\Delta}-1}}$

Upper bound: $mg\left(1-\frac{x_0}{D}\right)\sqrt{\left(\frac{x_0}{\Delta}\right)^2+2\frac{l}{\Delta}+2\sqrt{2\frac{l}{\Delta}}+1}$

g force felt by person during a leash fall:

Lower bound: $\sqrt{2\frac{l}{\Delta}-1}+1$

Upper bound: $\sqrt{2\frac{l}{\Delta}}+1$

Total drop of a leash fall:

Lower bound: $\sqrt{2l\Delta-\Delta^2}+\Delta+l$
Upper bound: $\sqrt{2l\Delta}+\Delta+l$

4.2 Observations

All of the expressions for tensions listed above are directly proportional to mg, the weight of the load. Hence, in the graphs below, we have chosen to present these tensions as simple numbers, which must be multiplied by a person's weight in order to find the actual tension that person would put on a slackline in that scenario. Similarly, the two expressions for the maximum acceleration are completely independent of the mass of the load, and are proportional to g. For this reason, in the list of formulas above and the graphs below, we have chosen to divide these expressions by g. This literally gives the g force that the person will feel during a leash fall.

In all of these expressions, all of the length measurements are relative. (Note that in the final form of each expression, every variable representing a length or distance appears in a fraction with another such unit.) This implies that we can consider all lengths to be measured relative to one particular length, say D, the distance between the two anchors. For example, if a person is standing 3 meters out on a slackline with a 10-meter span and the line sags 0.5 meters, then the tensions in the line would be exactly the same as if that person were standing 6 meters out on a 20-meter line and the line sagged 1 meter.

A pleasant side efect of this is that the units used to measure lengths do not matter at all, provided that the same units are used for all lengths. If you prefer to measure all your lengths in meters, but your weight in pounds, the formulas presented here will work for you with no modifications, and will return tensions measured in pounds. If you are a climber and prefer to think of forces in kN, but you think in feet rather than meters, you can convert your weight to kN, and leave all the length measurements in feet, and all the tensions will come out in kN.

4.3 Graphs

Figure 3 gives the tension in a slackline when a person is just standing on it. The different curves correspond to various values of x_0, the horizontal distance of the person from the anchor. These are all measured relative to

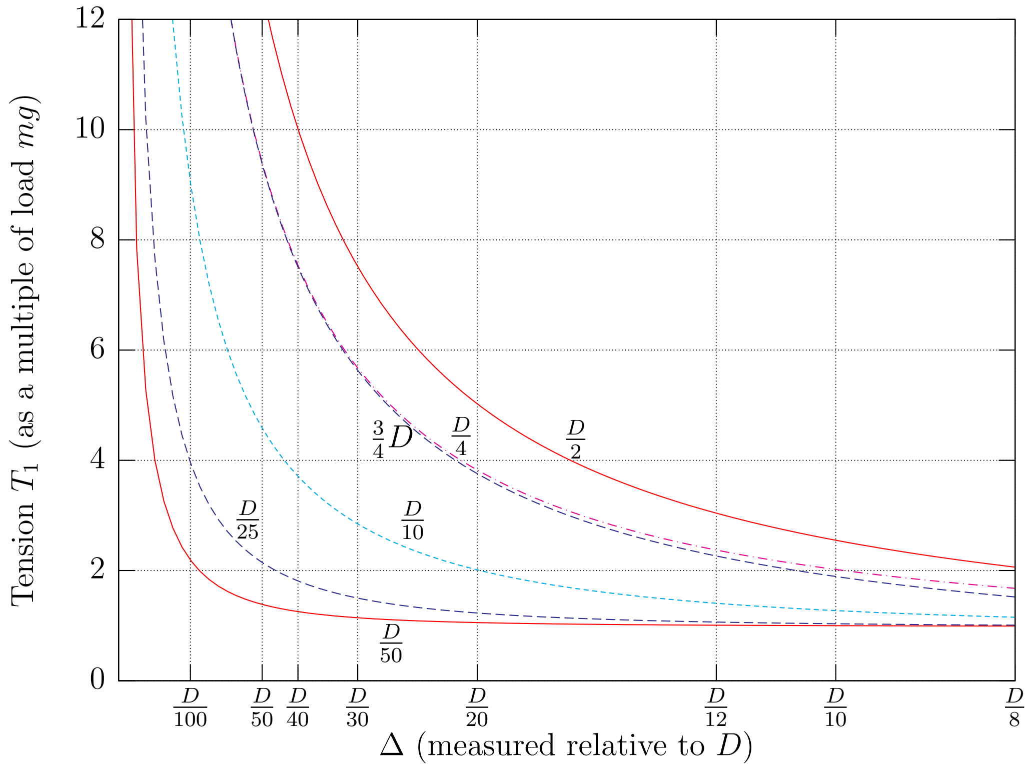

Figure 3: Tension in a slackline under a static load

D, as are the values of Δ along the horizontal axis. The graph makes it appear that the tension is highest when the person is in the middle of the line, and is much lower near the anchors, but this is misleading; keep in mind that Δ will be much higher in the middle of the line, and will be very small near the anchors. The graph does point out one interesting thing: the fact that the curves for $\frac{D}{4}$ and $\frac{3}{4}D$ are almost identical implies that when the person is standing $\frac{1}{4}$ of the way out from one anchor, the tensions on either side of the line are nearly the same (because Δ will certainly be the same at $x_0 = \frac{D}{4}$ and $x_0 = \frac{3}{4}D$).

Figure 4 shows that in a leash fall with $l = 8$ ft, g forces above $4g$ are unlikely unless Δ is less than 1 foot. For long slacklines, this usually only occurs near the anchors, but for short, tight highlines, it means that leash fals are going to hurt! It also shows that, unless the line sags a lot under body weight, a

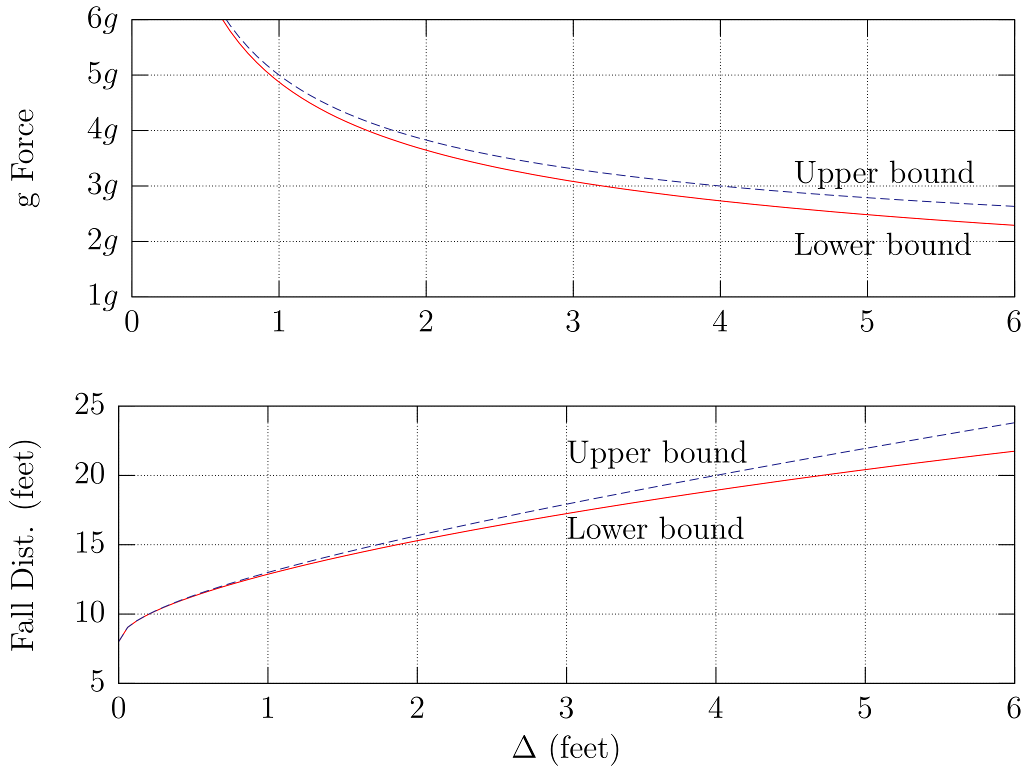

Figure 4: Max g force and drop distance (below anchors) during a leash fall with $l = 8$

leash fall shouldn't drop a person more than 25 feet. Recall that these two quantities depend only on l and Δ, so Figure 4 applies to a line of any length.

Figure 5 may be the most interesting (and perhaps shocking) in this section. It shows the maximum tension in one side of a 50-foot slackline (i.e. the maximum load put on one of the anchors) in the event of a leash fall, compared to the tension in the line right before the fall when the person was just standing. When the fall occurs in the middle of the line, the difference between the static load and the maximum tension is only about 0.5 times body weight, and when the fall occurs 10 feet from the anchor, it's only about 1 times body weight. Only when the leash fall occurs very close to the anchor is the tension significantly higher, and even then it's "only" a difference of 3 to 4 times body weight (2 to 3 kN for an average-sized person).

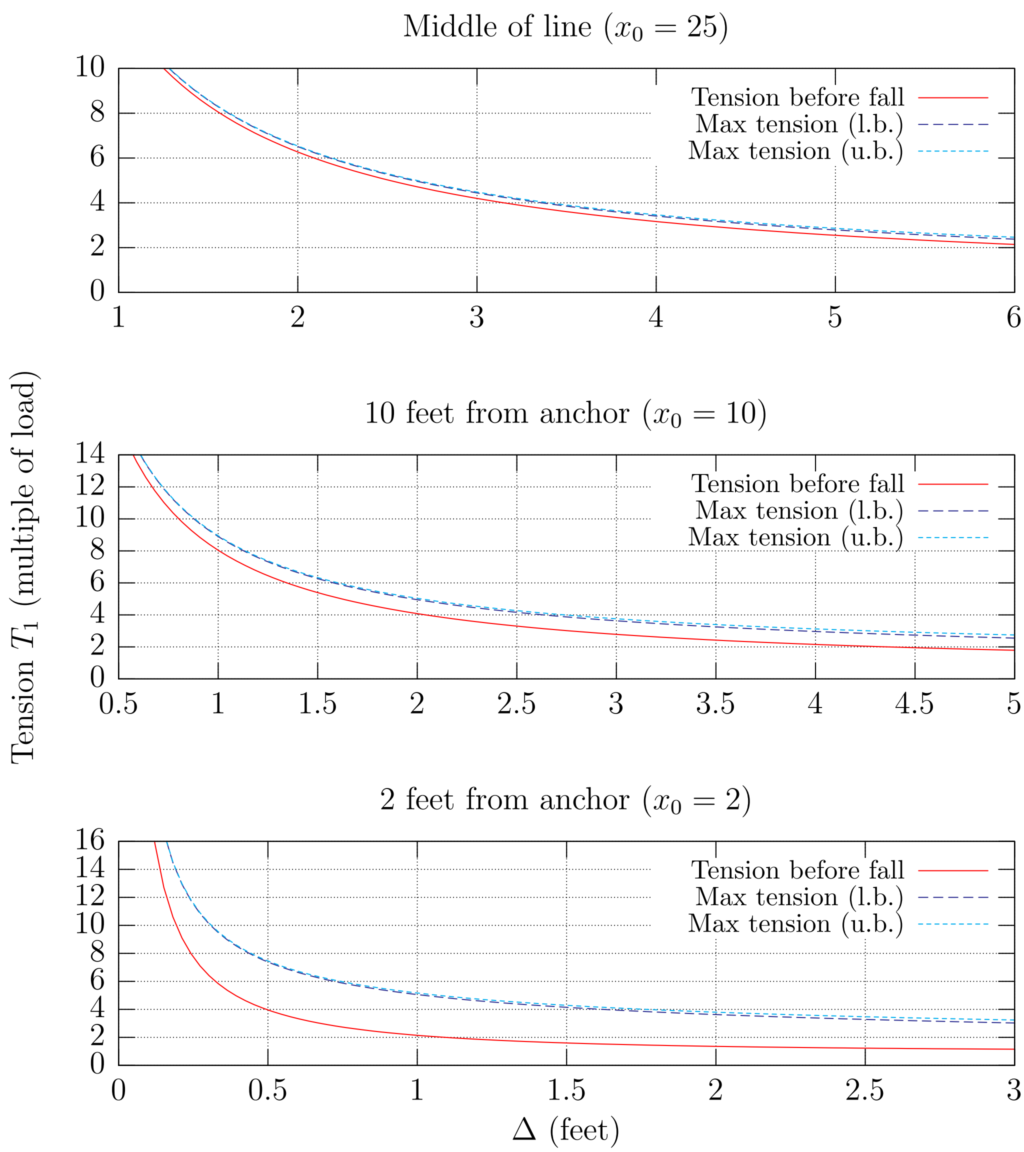

Figure 5: Tensions in a 50-foot slackline during a leash fall with $l = 8$

These graphs also show that the lower and upper bounds that we established earlier are quite close together, which is good.

5 Further Analysis

5.1 Linearity Revisited

The graphs presented in the previous section point out a significant weakness of the results presented thus far. With the linearity assumption made in section 2.2, we can do all of these calculations using a value of Δ measured at a particular point on a particular line. But Δ varies significantly from one point on a slackline to another (e.g. the line sags a lot more in the middle than near the anchors), and we still have no way to calculate this Δ at various points on the line.

Fortunately, it is possible to partially remedy this, using our linearity assumption once again. Suppose that we can measure Δ in the middle of the line. Then if the webbing obeys the linearity assumption of section 2.2, we can use this value to compute Δ at any other point on the line, and hence to compute all the other quantities derived above. From (6) in section 2.2, it is clear that in the middle of the line (when $x_0 = \frac{1}{2}D$) this value of Δ is

$$\Delta_m = \frac{1}{4}L \cdot \frac{mg}{K}.$$

(The m subscript here is for "middle", or for "maximum", since Δ is always greatest in the middle of the line.) Thus $L \cdot \frac{mg}{K} = 4\Delta_m$, so using (6) again, we see that at any other point on the line

$$\Delta = 4\Delta_m \cdot \frac{x_0}{D}\left(1 - \frac{x_0}{D}\right). \tag{10}$$

In section 3.3, we discussed why the linearity assumption of section 2.2 might be valid for our purposes, since many known nonlinearities of nylon webbing seem to occur over a long period of time (minutes at least) whereas a leash fall occurs in a split second. But now suppose we have set up a slackline, stretched it to the desired tightness, walked on it a few times and re-tightened as necessary, until it seems that, at least for the time being, the line sags the same amount in the middle each time it is weighted by the same load. (Experienced slackliners will agree that this is generally the way it goes.) This would seem to imply that, for the time being, the Young's modulus of the webbing is not changing, and thus that our linearity assumption might be valid.

To summarize, if we accept the linearity assumption, this gives us a new practical approach to calculating the tensions in a slackline and the other quantities discussed earlier. A person stands in the middle of the slackline and measures Δ_m (how much the line sags). This can then be used in equation (10) to predict what Δ should be for that person at any other point on the line, and this information can then be used with the equations in section 4.1 to calculate everything else. Furthermore, note that Δ_m is directly proportional to mg, so if this quantity is measured for one person, it can be calculated for any other person using their weight.

We will not bother to write down expressions for any of the quantities in section 3 in terms of Δ_m, because they are not too pretty. However, knowing the tension in the line under a static load is useful, and its expression is not that ugly. Plugging (10) into the results of section 2.1 gives

$$T_1 = mg\sqrt{\left(\frac{D}{4\Delta_m}\right)^2 + \left(1 - \frac{x_0}{D}\right)^2}.$$

Note that this implies that the force on an anchor increases as the person moves closer to the anchor. Fortunately, since it should be true on any real-world slackline that $\frac{D}{4\Delta_m} > 2$, the force is not tremendously higher; in fact, it varies by less than 12% over the length of the line.

5.2 More Graphs

Figure 6 shows Δ as a function of x_0 for various values of Δ_m. Note that the vertical axis is reversed, so in effect, this shows the exact curve that a person's feet should follow as they walk across a slackline. This graph is based on a 50-foot slackline, but all the lengths are relative as usual, so the numbers portrayed in the graph could be adapted to a line of any length.

Figure 7 shows two things: that the tension in the line is almost constant regardless of where the person is standing on the line (as explained in the previous section), and that the tensions seen during a fall (and hence the

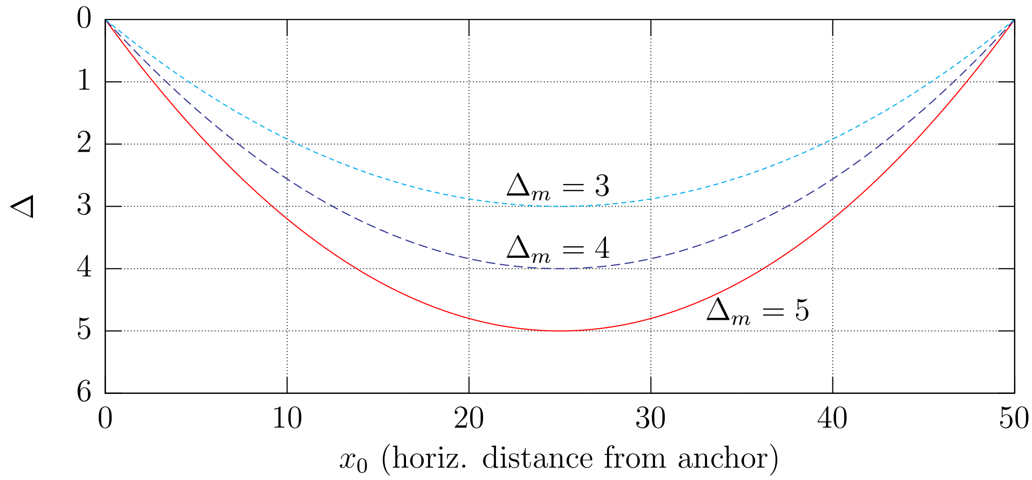

Figure 6: Sag in the line (Δ) as a function of position

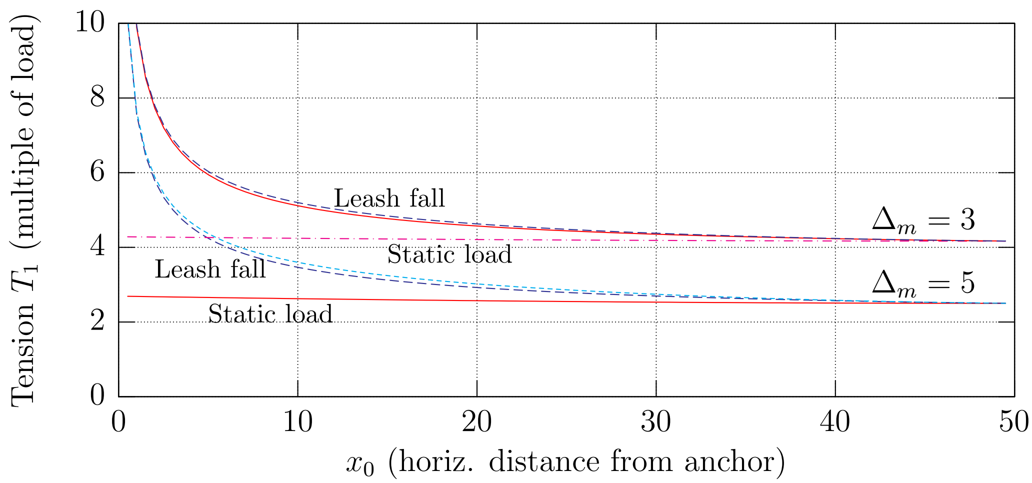

Figure 7: Tensions in a slackline before and during a leash fall

forces put on the anchors during a fall) are not much greater, except when the fall occurs very close to the anchor. This figure once again uses the values $D = 50$ and $l = 8$ (i.e. a 50-foot slackline and, perhaps, a 6-foot tall person using a 5-foot leash.)

6 Conclusions

We have found that by assuming that a slackline, once it has been set up and stretched appropriately, is linearly elastic, it is possible to calculate several important quantities based only on easily observable geometric parameters of the system. These calculations imply, among other things, that the load put on an anchor during a leash fall is not much higher than the load placed on the anchor by simply walking on the line, *except when the fall occurs very close to the anchor (within a few feet).*

The important unknown at this point is to what extent the assumption of linearity is valid. The results provided here should allow one to test this assumption in various ways. In particular, equation (10) gives us a very easy way to test this assumption experimentally. We can set up a slackline, and measure the value of Δ as we walk across it. If the values are fairly close to what equation (10) would predict, then we can conclude that, at least in the absence of drastic or sudden changes in the strain of the line, the assumption is more or less valid. The author intends to perform this experiment at some point in the future and report his findings, and he invites others to do the same.

The results of the leash fall models could be tested directly as well, using a load cell and the right testing apparatus. Regardless of the validity of the results presented here, it would be a Very Good Thing for the slacklining community to know the results of this sort of testing, so that we would know the kind of forces our anchors need to withstand, particularly in highlining scenarios.

One final point is that the calculations for leash falls done here were based on a completely static leash. In the real world, the leashes used in highlining are somewhat dynamic, and when a fall occurs very close to an anchor, this should reduce the load placed on that anchor. It is possible that, if the leash is dynamic enough, this may reduce the load significantly, but that would be the subject of another study.