Consideration of damping therefore results in complex eigenvalues and normally in complex eigenvectors. Each eigenvalue is therefore a pair of conjugate complex numbers. For technical applications only one eigenvalue of that pair is of importance. The real part contains information on the damping of the corresponding eigenvalue. The imaginary part one can derive the natural frequency

Computational cost is significantly larger than for a undamped modal analysis, since complex numbers have to be considered and the size of system matrices is double. For typical small values of damping the eigenvalues respectively natural frequencies are nearly independent of damping, such that damping is only seldom considered in modal analyses.

For the one degree-of freedom system on can derive the following relationship between decay constant δ and damping ratio ξ:

$$\xi = \frac{\delta}{\omega_0} = \frac{\delta}{2\pi f_0} \tag{5.3.17}$$

Thereby ω_0 and f_0 are the values of the undamped system.

The natural frequency f_D of the damped system can be derived from the undamped natural frequency f_0 and the damping ratio ξ

$$f_D = f_0 \cdot \sqrt{1-\xi^2} \tag{5.3.18}$$

Using a so-called proportional damping (e.g.. Rayleigh-damping with α and β, see section 2) also for the N degree-of freedom system it holds for each natural frequency

$$f_D = f_0 \cdot \sqrt{1-\frac{1}{4}\left(\frac{\alpha}{2\pi f_0}+2\pi f_0 \beta\right)^2} = f_0 \cdot \sqrt{1-\xi^2} \quad \text{with} \quad \xi = \frac{\alpha}{4\pi f_0}+\beta \pi f_0 \tag{5.3.19}$$

Based on the proceeding equations one can see, that for minor damping the influence on natural frequencies is small and quite simple calculated with equation (5.3.19). The damped eigenvectors are identical to the undamped eigenvectors if for the damping matrix holds:

$$C = M \cdot \sum_{i=1}^{n} a_i \left(M^{-1} K\right)^{i-1} \tag{5.3.20}$$

For n=2 one can derive the well known Rayleigh damping $C = a_1 M + a_2 K$, with that a costly modal analysis with consideration of damping is not needed.

The simple equation (5.3.18) is not longer applicable, if equation (5.3.20) is not fulfilled and an arbitrary damping is used. This is the case if only a single discrete damper in a N mass system is used or if the damping factors are not applied to the whole stiffness matrix but only to portions of it. In that case one can not calculate the influence of damping on natural frequencies independently, which results in time-dependent complex eigenvectors.

Another application of eigenvalue problems with damping is the consideration of gyro effects in rotors. Suppose a structure is spinning around an axis *r*. If a rotation about an axis perpendicular to *r* is applied to the structure, then a reaction moment appears. It is called the gyroscopic moment. Its axis is perpendicular to both the spinning axis *r* and the applied rotation axis.

The gyroscopic effect is thus coupling rotational degrees of freedom which are perpendicular to the spinning axis.

The additional gyroscopic matrix C_{gyr} takes the same position in the differential equation as the damping matrix and has to be treated in the same fashion.

$$M\ddot{u} + \left(C + C_{gyr}\right)\dot{u} + K u = 0 \qquad (5.3.21)$$

E.g. the gyroscopic damping matrix of the ANSYS element PIPE16 looks like:

$$[C_e] = 2\Omega\rho AL \begin{bmatrix} 0 & & & & & & & & & & & \\ 0 & 0 & & & & & & \text{Antisymmetric} & & & & \\ 0 & -g & 0 & & & & & & & & & \\ 0 & 0 & 0 & 0 & & & & & & & & \\ 0 & -h & 0 & 0 & 0 & & & & & & & \\ 0 & 0 & -h & 0 & -i & 0 & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\ 0 & 0 & -g & 0 & -h & 0 & 0 & 0 & & & & \\ 0 & g & 0 & 0 & 0 & -h & 0 & -g & 0 & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\ 0 & -h & 0 & 0 & 0 & j & 0 & h & 0 & 0 & 0 & \\ 0 & 0 & -h & 0 & -j & 0 & 0 & 0 & h & 0 & -i & 0 \end{bmatrix}$$

Since the gyroscopic effect is no damping in the strict sense, the real part of the eigenvalue is zero. Nevertheless there are complex eigenvectors.