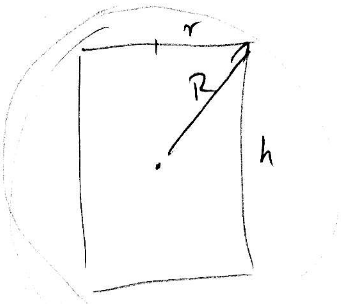
Zylinder + umbeschriebene Kugel



$$R = \sqrt{r^2 + \frac{h^2}{4}}$$

$$V_{Kugel} = \frac{4\pi}{3} R^3 = \frac{4\pi}{3}\left(r^2 + \frac{h^2}{4}\right)^{3/2}$$

$$V_{Zylinder} = \pi r^2 \cdot h$$

Verhältnis $V_Z / V_K = f$

$$f = \frac{\pi r^2 \cdot h}{\frac{4\pi}{3}\left(r^2 + \frac{h^2}{4}\right)^{3/2}}$$

Sei allgemein $h = x \cdot r$

$$f = \frac{3 r^2 \cdot x r}{4\left(r^2 + \frac{x^2 r^2}{4}\right)^{3/2}} = \frac{3x \cancel{r^3}}{4 \cancel{r^3}\left(1 + \frac{x^2}{4}\right)^{3/2}} = \frac{3x}{4\left(\frac{x^2}{4}+1\right)^{3/2}}$$

Grenzfall $x \to 0$ $\qquad x \to \infty$

$f \to 0$ $\qquad f \to 0$

Maximum von f: $$\frac{df}{dx} = \frac{4\left(\frac{x^2}{4}+1\right)^{3/2} \cdot 3 - 3x \cdot 4 \cdot \frac{3}{2}\left(\frac{x^2}{4}+1\right)^{1/2} \cdot \frac{x}{2}}{16\left(\frac{x^2}{4}+1\right)^3}$$

$= 0$

$$\Rightarrow \cancel{12}\left(\frac{x^2}{4}+1\right)^{\cancel{3/2}} = \cancel{12}x \cdot \frac{3}{2} \cdot \frac{x}{2} \cancel{\left(\frac{x^2}{4}+1\right)^{1/2}}$$

$$\frac{x^2}{4} + 1 = \frac{3}{4}x^2$$

$$\cancel{\frac{1}{4}x^2 = 1 \quad x^2 = 4 \quad \rightarrow x = 2}$$

2/4*x^2 = 1 --> x = Wurzel(2)