MEASUREMENT OF MIXED-MODE DELAMINATION FRACTURE TOUGHNESS OF UNIDIRECTIONAL GLASS/EPOXY COMPOSITES WITH MIXED-MODE BENDING APPARATUS

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Abstract
Initiation of cracking and delamination growth in a unidirectional glass/epoxy composite were evaluated under mode I, mode II, and mixed mode I + II static loading. They have been expressed in terms of the total critical strain energy release rate, \( G_{IC} \), and the total fracture resistance, \( G_{TR} \). For the mixed mode I + II, a mixed-mode bending apparatus was used. The loading in this test is a simple combination of the double cantilever beam mode I and the end notch flexure mode II tests. In addition to characterisation of delamination initiation, whatever the value of the \( G_{II}/G_T \) modal ratio, this apparatus allows the plotting of an \( R \) curve, from which we obtain the total fracture resistance, \( G_{TR} \).

Experimental results were correlated with computations of a semi-empirical criterion through the plotting of the total critical strain energy release rate, \( G_{IC} \), versus the \( G_{II}/G_T \) modal ratio, and of the total fracture resistance, \( G_{TR} \), versus the \( G_{II}/G_T \) modal ratio.

Keywords: mode I, mode II, mixed mode, strain energy release, \( R \) curve, semi-empirical criterion

NOTATION
- \( a_{eff} \): Effective crack length
- ALDCB: Asymmetrical load double cantilever beam
- \( B \): Width of specimen
- \( C \): Compliance
- CBON: Cantilever beam opening notch
- CLS: Cracked lap shear
- DCB: Double cantilever beam
- EDT: Edge delamination tension
- ELS: End loaded split
- ENF: End notch flexure
- \( G_{IC}, G_{HC}, G_{TC} \): Mode I, mode II and total critical strain energy release rate
- \( G_{IR}, G_{HR}, G_{TR} \): Mode I, mode II and total fracture resistance

1 INTRODUCTION
The delamination process is frequently met in composite materials and in most cases it results from mode I, mode II, or mixed mode I + II delamination. Delamination failure under pure mode I, where DCB testing is commonly used and under pure mode II where ENF and ELS tests are frequently used, have been well studied. However, in most realistic situations composite structures are subjected to both modes listed above. For the mixed mode, different specimens have been proposed in the literature, most of which present practical limitations in their use. In fact, some do not permit testing of materials in a large range of mixed-mode ratios, \( G_{II}/G_T \), such as ALDCB, MMF, CBON whereas for others, either it is difficult to interpret results, making it necessary to use a finite element analysis such as CLS and EDT, or requiring the use of complex machines (ALDCB). For instance, we may mention the case of the IDCB test, for which it is impossible to plot the \( R \) curve, since the mixed-mode ratio depends on crack length. Consequently, the MMB test appears to be very interesting for testing materials in mixed mode because it allows characterisation of the delamination initiation and growth for any value of the \( G_{II}/G_T \) modal ratio.

The present study concerns the characterisation of delamination initiation and growth on the basis of a strain energy release rate concept. In addition to
mode I and mode II, we have studied six $G_{II}/G_T$ modal ratios. For each $G_{II}/G_T$ ratio several specimens were tested. Good agreement was obtained between experimental results for $G_{TC}$ and $G_{TR}$, and calculations of a semi-empirical criterion. For the $G_{TC}$ versus $G_{II}/G_T$ modal ratio we found $m_{TC} = 2.6$, but for the $G_{TR}$ versus $G_{II}/G_T$ modal ratio we found $m_{TR} = 1$. The latter corresponds to the classical criterion.

2 RESULTS AND DISCUSSION

2.1 Material
The material used in this study was an M10 epoxy resin (Vicotex) reinforced with 52% by volume of E-glass fibres, 5% of which are woven perpendicularly. This material was prepared in the form of compression-moulded, 6 mm thick panels. The crack starter was formed by inserting a Teflon film at mid-thickness during moulding.

The elastic constants obtained experimentally were:

- $E_{11} = 36.2$ GPa
- $E_{22} = 10.6$ GPa
- $E_{33} = 7.2$ GPa
- $G_{12} = 5.6$ GPa
- $G_{13} = 3.7$ GPa
- $G_{23} = 3.2$ GPa
- $\nu_{12} = 0.26$
- $\nu_{13} = 0.33$
- $\nu_{23} = 0.48$

2.2 Specimens
The following test specimens were subjected to monotonic flexural loading:

- In pure mode I, a DCB (Fig. 1(a)).
- In pure mode II, an ELS (Fig. 1(b)).

In mixed mode, an MMB specimen proposed by Crews et al. and adapted to our material by Aboura et al. based on the work of Reeder et al. This is a simple combination of a DCB (mode I) specimen and an ENF (mode II) specimen (Fig. 2). Load is applied to a split-beam specimen by means of a lever where the distance, $e$, between the load point and the fulcrum can be varied. The design of the MMB apparatus allows us to introduce mode I loading at the end of the lever and mode II loading at the fulcrum.

2.3 Test conditions
All tests were performed in an Instron machine at 2 mm/min constant displacement rate. In order to study the damage initiation mechanisms and crack propagation, a strain gauge was bonded to each specimen. An acquisition system permits simultaneous recording of the load, $P$, the displacement, $\delta$, the response of the strain gauge and the acoustic emission from the loaded sample.

2.4 Measurement of $G_{TC}$ and $G_{TR}$
The total critical strain energy release rate, $G_{TC}$, was determined experimentally by the Irwin–Kies compliance method:

$$G_{TC} = \frac{P_{TC}^2 dC}{2B \, da} \quad (1)$$

2.4.1 Compliance
The compliance of a composite material in mode I when using a DCB specimen is given by $C = a^n/h$, where $n$ and $h$ are empirical constants for the laminates under consideration.

In the case of mode II and mixed-mode I + II, the compliance is $C = a + \beta a^2$, where $\alpha$ and $\beta$ are empirical constants that depend on the $G_{II}/G_T$ modal ratio. The compliances for different $G_{II}/G_T$ modal ratios considered in this study were computed (Fig. 3). We have observed that the compliance increases when the $G_{II}/G_T$ modal ratio decreases.

Fig. 1. (a) DCB specimen; (b) ELS specimen.

Fig. 2. MMB apparatus.
2.4.2 Total critical energy, $G_{TC}$

The simultaneous recording of the load, the displacement of the load application point, the response of the strain gauge, and the acoustic emission allows us to detect precisely the critical point, $A$ ($P_{TC}, \delta_{TC}$), which is necessary to determine $G_{TC}$. Many studies\(^5,9,19\) have shown that this method is more useful than 'visual assessment' (as defined in the new ASTM standard 05528-94a).

Table 1 summarises the results of the $G_{TC}$ values as

<table>
<thead>
<tr>
<th>$e^*$ (mm)</th>
<th>$\Delta U/\Delta h$ (%)</th>
<th>$a_0$ (mm)</th>
<th>$P_{TC}$ (N)</th>
<th>$G_{TC}$ (J/m$^2$)</th>
<th>$e$ (mm)</th>
<th>$\Delta U/\Delta h$ (%)</th>
<th>$a_0$ (mm)</th>
<th>$P_{TC}$ (N)</th>
<th>$G_{TC}$ (J/m$^2$)</th>
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<tbody>
<tr>
<td>Mode II</td>
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<tr>
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<td>327.7</td>
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Table 1. Results of strain energy release rates $G_{TC}$, $G_{IC}$, and $G_{TC}$

$G_{TC}$ = 2905.76 ± 224.55 (J/m$^2$) $G_{TC}$ = 579.62 ± 58.66 (J/m$^2$)

| $G_{IC}$ = 2457.26 ± 100.30 (J/m$^2$) $G_{IC}$ = 568.36 ± 98.58 (J/m$^2$) |
|------------------------|-------------|----------------|
| 30                     | 91          | 23             |
| 30                     | 91          | 24.5           |
| 30                     | 91          | 25             |
| 30                     | 91          | 25.5           |

$G_{TC}$ = 1821.93 ± 84.47 (J/m$^2$) $G_{IC}$ = 340.35 ± 37.26 (J/m$^2$)

$G_{TC}$ = 1033.67 ± 174.11 (J/m$^2$) $G_{IC}$ = 118.02 ± 37.22 (J/m$^2$)

\(^*\) See Fig. 2.
a function of crack length and $G_{II}/G_T$ modal ratio. The results show a logical development. The higher the $G_{II}/G_T$ modal ratio, the greater the value of $G_{TC}$.

We have observed that there is a large difference between values of $G_{IC}$ and of $G_{IIIC}$; this corresponds specifically to the behaviour of composites consisting of a brittle matrix.  

2.4.3 Total fracture resistance, $G_{TR}$

The different compliance laws corresponding to $G_{II}/G_{TC}$ modal ratio values considered in this study allowed us to plot the $R$ curves. These show the variation in total fracture resistance, $G_{TR}$, as the delamination extends.

The following equations permit computation of $a_{eff}$. From the compliance, $C$, for mode I:

$$C = \frac{a_{eff}^{n}}{h} = \frac{\delta_{IR}}{P_{TR}}$$

which yields:

$$a_{eff} = \left( \frac{h}{P_{TR}} \right)^{1/n}$$  \hspace{1cm} (2)

For mode II and mixed mode:

$$C = \alpha + \beta a_{eff}^{3} = \frac{\delta_{TR}}{P_{TR}}$$

From these we obtain:

$$a_{eff} = \left( \frac{\delta_{TR} - \alpha}{\beta} \right)^{1/3}$$  \hspace{1cm} (3)

Thus, the total fracture resistance during crack growth is expressed as:

Mode I:  

$$G_{TR} = \frac{n P_{TR}^{2} a_{eff}^{n-1}}{2Bh}$$  \hspace{1cm} (4)

Mode II and mixed mode:  

$$G_{TR} = \frac{3 P_{TR}^{4} \beta a_{eff}^{2}}{2Bh}$$  \hspace{1cm} (5)

The $R$ curve method allows the strain energy release rate to increase with crack length, reaching a plateau for $a_{eff} \geq 50$ mm (Fig. 4) corresponding to $G_{II}, G_{HR}$ and $G_{TR}$, as appropriate.
Figure 4 shows examples of $R$ curves for two mixed-mode ratios, $G_{II}/G_T = 28$ and 43%, obtained from eqns (2)-(5).

The variation of $G_{TR}$ with $G_{II}/G_T$ modal ratio is related to the fracture mechanisms dissipating the energy taken into account according to the $G_{II}/G_T$ modal ratio considered. Before interpreting the rupture mechanisms it is interesting to show the changes in the load/displacement curves which progressively change from the mode I curve form to the mode II curve form, according to the $G_{II}/G_T$ modal ratio value (Fig. 5). In fact, for pure mode I the load increases continually from the point of damage initiation, $A$ (Fig. 5(a)), and then reaches a plateau where the load is almost constant. At $G_{II}/G_T = 28\%$, the load increases in a single stage before starting a

![Fig. 5.](image-url)
progressive decrease with a low slope (Fig. 5(b)). In the case of $G_{II}/G_T = 43\%$, this load decrease becomes more pronounced after the damage initiation point (Fig. 5(c)). When the participation rate of mode I and mode II are the same ($G_{II}/G_T = 53\%$), we notice the appearance of a slight instability of the load after the damage initiation point characterising a more substantial participation of mode II compared to previous cases (Fig. 5(d)). For $G_{II}/G_T = 72\%$ (Fig. 5(e) and (f)), an important drop in the load occurs just after the point of damage initiation. This is a characteristic of the propagation instability phenomenon non noticed generally in pure mode II (Fig. 5(g)). This propagation instability raises the difficulty of the total fracture resistance measurement as we approach pure mode II. For the $G_{II}/G_T = 82\%$ case, all the curves exhibit drastic drops after initiation of the delamination process, which characterises unstable propagation. In this case, if we consider $G_{TR} = G_{TC}$ we note that we underestimate the total fracture resistance compared to the other modal ratios that we studied. There is only one curve that presents a plateau.
permitting measurement of the total fracture resistance for which the value is acceptable.

Table 2 lists the experimentally obtained results for \( G_{\text{IR}}, G_{\text{II}}, \) and \( G_{\text{TR}}. \)

### 2.5 Fractographic analysis

In order to understand the interlaminar fracture mechanisms, the fracture surfaces were examined in a scanning electron microscope (SEM). It was observed that pure mode I is characterised by fractures localised principally in the resin and along the resin/fiber interface (Fig. 6). By contrast, pure mode II is characterised by fractures localised in the resin with many hackles having an orientation of approximately 45° with respect to the fiber direction (Fig. 7), which for mode II loading is the principal normal stress plane.\(^{21}\)

In the case of mixed mode, the mechanisms are more complex.\(^{22,23}\) Figure 8 shows a fracture surface corresponding to \( G_{\text{II}}/G_{\text{T}} = 28\% \). As we see, the

<table>
<thead>
<tr>
<th>( e^* ) (mm)</th>
<th>( G_{\text{II}}/G_{\text{T}} ) (%)</th>
<th>( G_{\text{TR}} ) (J/m(^2))</th>
<th>( e ) (mm)</th>
<th>( G_{\text{II}}/G_{\text{T}} ) (%)</th>
<th>( G_{\text{TR}} ) (J/m(^2))</th>
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<td>53</td>
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<tr>
<td>( G_{\text{II}} = 2905.76 \pm 244.55 ) (J/m(^2))</td>
<td>( G_{\text{TR}} = 1752.60 \pm 129.73 ) (J/m(^2))</td>
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<td>( G_{\text{TR}} = 2457.26 \pm 100.30 ) (J/m(^2))</td>
<td>( G_{\text{TR}} = 1439.50 \pm 140.29 ) (J/m(^2))</td>
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<td>40</td>
<td>72</td>
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<tr>
<td>( G_{\text{TR}} = 2625.00 \pm 0 ) (J/m(^2))</td>
<td>( G_{\text{TR}} = 1118.37 \pm 111.07 ) (J/m(^2))</td>
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\( ^{a} \) See Fig. 2.
The fracture surface exhibits cleavage paths characterising the opening mode. It also shows hackles oriented at less than 45° with respect to the fracture surface. These hackles are mainly forward as a consequence of matrix shear rupture caused by the participation of mode II. Some of the hackles are fractured by mode I at their base on the rupture surface (Fig. 9). This results in the presence of traces displaying a river-like form. As the participation of mode II increases, more developed shear hackles are observed and the cleavage paths are less frequent (Fig. 10).

2.6 Reflection on the application of a semi-empirical criterion

In order to predict $G_{TC}$ and $G_{TR}$ as functions of $G_{II}/G_T$ modal ratio, the following relationship is suggested:

$$G_{TC} = G_{IC} + (G_{II} - G_{IC})\left(\frac{G_{II}}{G_T}\right)^{m}$$

On the other hand, it is interesting to know how $G_{TR}$ increases independently of $G_{TC}$ as a function of $G_{II}/G_T$. The experimental measurements of $G_{TC}$ are thus presented as a function of $G_{II}/G_T$ modal ratio in Fig. 11. Figure 12 shows the mode I energy contribution versus the mode II energy component. The application of this semi-empirical criterion gives good results for $m = 2.6$:

$$G_{TC} = 118.31 + 2795.18\left(\frac{G_{II}}{G_T}\right)^{2.6}$$
From experiment: $G_{IC} = 118.02 \text{ J/m}^2$; $G_{IIc} = 2905.76 \text{ J/m}^2$; $G_{IIIc} - G_{IC} = 2787.74 \text{ J/m}^2$. We see that the curve $G_{TC} = f(G_{II}/G_T)$ is composed of three stages:

- a first stage, $0\% \leq G_{II}/G_T \leq 53\%$, where $G_{TC}$ increases progressively when the $G_{II}/G_T$ modal ratio increases;
- a transition stage, $53\% \leq G_{II}/G_T \leq 72\%$, where the curve takes a large radius towards higher energy values than in previous cases; this is due to the fact that the mode II ratio is becoming important;
- a final stage, $72\% \leq G_{II}/G_T \leq 100\%$, where the energy values were comparable to the pure mode II values.

Practically the same $m$ parameter value ($m_2 = 2$) has been obtained with experimental IDCB results. This method does not permit the measurement of $G_{IR}$ because the $G_{II}/G_T$ ratio depends on the crack length. For this purpose, we have used the MMB apparatus.

In Fig. 13 are plotted values of $G_{TR}$ versus the $G_{II}/G_T$ modal ratio. The correlation between the experimental values of $G_{TR}$ and the semi-empirical calculations appears to be better than the case of $G_{TC}$ with $m_R = 1$, and have used the classical criterion used widely in the literature:

$$G_{TR} = 419.21 + 2408.34 \left( \frac{G_{II}}{G_T} \right)^{1}$$

From experiment: $G_{IR} = 428.75 \text{ J/m}^2$; $G_{IIIR} = 2905.76 \text{ J/m}^2$; $G_{IIIR} - G_{IR} = 2477.01 \text{ J/m}^2$.

Figure 14 shows the mode I fracture resistance contribution versus the mode II component. It appears to us that the $m$ parameter ($m_1$, $m_R$) permits us to distinguish between the changes in $G_{TC}$ and $G_{TR}$ as functions of the $G_{II}/G_T$ modal ratio. In order to verify the generality of the criterion, studies on woven composites have shown that the $m$ parameter depends on whether the resin is brittle ($m_1 = 2$) or ductile ($m_1 = 3$), but it is independent of...
the displacement rate of loading. However the criterion provides a good fit to the delamination initiation results for carbon-fibre/epoxy composites, with \( m_1 \) taking the value of 1.557.

3 CONCLUSIONS

- The MMB apparatus appears to be very easy in use. It simulates perfectly the mixed-mode delamination, and permits computation of \( G_{TR} \) and therefore the \( R \) curve for any \( G_{II}/G_T \) modal ratio value.
- We have shown that the form of the curve is characteristic of the mixed-mode ratio considered.
- Good agreement was found between the experimental results and calculations of a semi-empirical criterion. In fact, \( G_{TC} \) increases as a function of \( G_{II}/G_T \) with \( m_1 = 2.6 \), whereas \( G_{TR} \) versus \( G_{II}/G_T \) modal ratio is linear with \( m_R = 1 \).

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